Adjusting USCF Ratings from FIDE events

USCF Ratings Committee

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This document describes a proposed update for the procedure to rate USCF ratings from the results of FIDE-rated events that are not normally rated by the USCF. The original procedure was described in the 1994 USCF Ratings Committee report, and can be obtained on the web at

`http://math.bu.edu/people/mg/ratings/report94.pdf`

There are two changes from the 1994 algorithm.

1. Because FIDE rates in periods of three months instead of six months, the USCF rating updates based on non-USCF-rated FIDE events are carried out every three months.

2. As a result of major changes in the USCF rating system in 2001, the formulas in the FIDE-updating procedure were changed accordingly.

Note that this procedure does not apply to players who are FIDE-unrated. Such players do not have their USCF rating updated. Also, the procedure only applies to players with USCF ratings based on at least 9 rated games, and who have not had either all wins and all losses.

1 The Algorithm

Identify the FIDE-rated events that were not USCF-rated (these should be the ones played outside the U.S.). Perform the following computations for the non-USCF events:

1. Calculate the total number of games competed in the last three months. Call this m.

2. Calculate the sum of the total scores. Call this value W.

3. Calculate the sum of the differences between the total score and expected score. Call this value d.

4. Compute the sum of expected scores, E, by subtraction:

$$E = W - d$$

5. Let A be the amount that needs to be added to the player's FIDE rating to place it on the USCF scale. This value is obtained from the conversion described in the rating system description for setting an initial USCF rating for FIDE-rated players. See *http://math.bu.edu/people/mg/ratings/rating.system.pdf* for more details.

6. Let R_f be the pre-period FIDE rating, and let R_u be the player's current USCF rating. Now compute the "adjusted" expected score, E_{adj} using the formula:

$$E_{adj} = \frac{m}{1 + 10^{(R_f - R_u + A)/400}(\frac{m}{E} - 1)}.$$

7. Now compute the player's updated rating using the formula

$$R_{new.u} = R_u + K(W - E_{adj})$$

with

$$K = \frac{400}{N_{eff} + m},$$

where N_{eff} is the effective number of games already played in USCF events, and is part of the computation of the player's USCF rating.

2 Justification for the algorithm

Step 6 in the algorithm, the adjusted expected score calculation, is derived from a straightforward logic, though this may not be apparent in the formula itself. The logic of deriving this formula follows these steps:

- Calculate the average FIDE rating of the opponents (derived from the total expected score and the winning expectancy formula)
- Convert the opponents' average FIDE rating to an average on the USCF scale
- Calculate the total expected score against the opponents' average (converted) USCF rating relative to the player's actual USCF rating.

The mathematical derivation proceeds as follows. With m the total number of FIDE games in the past three months, E the total expected score (from step 4), and R_f the player's pre-period FIDE rating, let $\bar{r}_{opp}$ be the (unobserved) average FIDE rating of the opponents. An estimate of $\bar{r}_{opp}$ can be obtained by recognizing that the total expected score over m games, E, is approximately equal to m times the winning expectancy against a single opponent with rating $\bar{r}_{opp}$, that is

$$E \approx m\left(\frac{1}{1+10^{(\bar{r}_{opp}-R_f)/400}}\right).$$

Solving for $\bar{r}_{opp}$ yields

$$\bar{r}_{opp} \approx 400\log_{10}\left(\frac{m}{E}-1\right)+R_f.$$

With A being the additive amount that places a FIDE rating onto the USCF scale, the average rating of the opponents on the USCF scale is approximately $(\bar{r}_{opp}+A)$. The total expected score on the USCF scale is approximately m times the winning expectancy against a single opponent with rating $(\bar{r}_{opp}+A)$ relative to the player's USCF rating, that is

$$\begin{aligned} E_{adj} &\approx m\left(\frac{1}{1+10^{((\bar{r}_{opp}+A)-R_u)/400}}\right) \\ &= \frac{m}{1+10^{(400\log_{10}(m/E-1)+R_f+A)-R_u)/400}} \\ &= \frac{m}{1+10^{(R_f-R_u+A)/400}\left(\frac{m}{E}-1\right)} \end{aligned}$$

The second line above replaces the formula for $\bar{r}_{opp}$ into the expression. The end result in the USCF-adjusted total expected score over m games.

3 Example calculation

Alexander Onischuk's performances in FIDE events played outside the US for the period of May – July 2004 can be obtained on the FIDE web site:

```
Onischuk, Alexander  Total change: 2.80

11th RUS Team Champ.Men      Dagomys  RUS 2004-04-20
  Rc    Ro     w  n  change   K   K*chg
2527  2652  5.5  8    0.14  10    1.40

5 A. Karpov Intl.      Poikovsky  RUS  2004-03-17
  Rc    Ro     w  n  change   K   K*chg
2679  2652  4.5  9    0.36  10    3.60
```

The algorithm to update Onischuk's USCF rating proceeds as follows (Onischuk's USCF rating in June 2004 was 2706):

1. Calculate the total number of games competed in the three month period:

$$m = 8 + 9 = 17$$

2. Calculate the sum of the total scores:

$$W = 5.5 + 4.5 = 10$$

3. Calculate the sum of the differences between the total score and expected score:

$$d = 0.14 + 0.36 = 0.50$$

4. Compute the sum of expected scores:

$$E = W - d = 10 - 0.50 = 9.5$$

5. Assume for the purposes of computation that the FIDE rating is 50 points higher than a USCF rating for a FIDE rating of 2652 (refer the USCF rating system document to determine this difference). Thus, the value of A is 50.

6. Compute the adjusted sum of expected scores:

$$E_{adj} = \frac{17}{1 + 10^{(2652-2706+50)/400}(\frac{17}{9.5} - 1)} = \frac{17}{1 + (0.9772)(0.7895)} = \frac{17}{1.7715} = 9.596.$$

7. Compute the new USCF rating. With a USCF rating over 2200, and having played more than 50 games, $N_{eff} = 50$ (see the USCF rating system specifications for details).

$$K = \frac{400}{N_{eff} + m} = \frac{400}{50 + 17} = 5.97.$$

   Thus

$$R_{new.u} = 2706 + 5.97 \times (10 - 9.596) = 2706 + 2.41 \approx 2708$$