The USCF Rating System

Prof. Mark E. Glickman
Boston University

Thomas Doan
Estima

June 4, 2009

The following algorithm is the procedure to rate USCF events. The procedure applies to two separate rating systems: the Quick Chess (QC) system, and the Regular system. The QC system governs events with time controls of G/5 through G/60. Regular events have time controls of G/30 or slower. The formulas apply to each system separately. Events having time controls between G/30 and G/60 are rated in both systems (i.e., dual-rated).

1 The Rating Algorithm

Before an event, a player is either unrated, or has a rating based on having played N games. A player's rating is termed "provisional" if it is based on 25 or fewer games, and is "established" otherwise. Assume the player competes in m games during the event. Post-event ratings are computed in a sequence of five steps:

- The first step sets temporary initial ratings for unrated players.
- The second step calculates an "effective" number of games played by each player.
- The third step calculates temporary estimates of ratings for certain unrated players only to be used when rating their opponents on the subsequent step.
- The fourth step then calculates intermediate ratings for all players.
- The fifth step uses these intermediate ratings from the previous step as estimates of opponents' strengths to calculate final post-event ratings.

The calculations are carried out in the following manner:

Step 1: Set initial ratings for unrated players.

Initial rating estimates are set for all unrated players in an event. The purpose of setting initial rating estimates for unrated players is (1) to be able to incorporate information about a game result against an unrated player, and (2) to choose among equally plausible ratings during a rating calculation for an unrated player (see the details of the "special" rating formulas in Section 2.2.

An initial rating for an unrated player is determined in the following order of precedence.

- If an unrated player has a FIDE rating, use a converted rating according to the following formula:

$$\text{USCF} = \begin{cases} 720 + 0.625 \times \text{FIDE} & \text{if FIDE} < 2000 \\ -350 + 1.16 \times \text{FIDE} & \text{if FIDE} \geq 2000 \end{cases}$$

  If the FIDE rating is over 2150, then this converted rating is treated as based on having played 10 games ($N = 10$). If the FIDE rating is 2150 or less, then this converted rating is treated as based on having played 5 games ($N = 5$).

- If an unrated player has a CFC rating over 1500, use a converted rating according to the following formula:

$$\text{USCF} = 1.1 \times \text{CFC} - 240.$$

  This converted rating is treated as based on having played 5 games ($N = 5$).

  If an unrated player has a CFC rating of 1500 or less, use a converted rating according to the following formula:

$$\text{USCF} = \text{CFC} - 90.$$

  This converted rating is treated as based on having played 0 games ($N = 0$).

  If a player has a foreign national rating, but no CFC, FIDE (or USCF) rating, the USCF office may at their discretion use a rating of their determination on a case-by-case basis. In such a case, the rating is treated as based on having played 0 games ($N = 0$).

- If the event is a regular event and the player has no regular rating, but has a QC rating based on at least four games, then use the QC rating as the imputed rating. This rating is treated as being based on 0 games ($N = 0$).

Conversely, if the event is a QC event and the player has no QC rating, but has a regular rating based on at least four games, then use the regular rating as the imputed rating. The rating is assumed to be based on the lesser of 10 and the number of games on which the regular rating itself is based ($N = 10$ or $N =$ prior number of regular games, whichever is smaller).

- Otherwise, impute an age-based rating according to the following procedure. Define a player's age (in years) to be

$$\text{Age} = (\text{Tournament End Date} - \text{Birth Date})/365.25.$$

The formula for an initial rating based on age is given by

$$\text{USCF} = \begin{cases} 50 \times \text{Age} & \text{if } 3 \leq \text{Age} \leq 26 \\ 1300 & \text{otherwise.} \end{cases}$$

The rating is assumed to be based on 0 games ($N = 0$). If an unrated player does not provide a birth date, but is inferred to be an adult (e.g., through an appropriate USCF membership type), then the initial rating is set to be 1300 with $N = 0$, treating the player as a 26-year old in the Age-based calculation. As a practical concern, if "Age" is calculated to be less than 3 years old, then it is assumed that a miscoding of the player's birthday occurred, and such a player is also treated as a 26-year old in the Age-based calculation.

- If no international rating or birth information is supplied, and if the player does not have a non-correspondence USCF rating, impute a rating of 750. This rating is assumed to be based on 0 games ($N = 0$).

Step 2: Calculate the "effective" number of games played by each player.

This number, which is typically less than the actual number of games played, reflects the uncertainty in one's rating, and is substantially smaller especially when the player's rating is low. This value is used in the "special" and "standard" rating calculations. See Section 2.1 for the details of the computation.

Step 3: Calculate a first rating estimate for each unrated player for whom Step 1 gives $N = 0$. For these players, use the "special" rating formula (see Section 2.2), letting the "prior" rating be R_0. However, for only this step in the computation, set the number of effective games for these players to 1 (this is done to properly "center" the ratings when most or all of the players are previously unrated).

- If an opponent of the unrated player has a pre-event rating, use this rating in the rating formula.

- If an opponent of the unrated player is also unrated, then use the initial rating imputed from Step 1.

If the resulting rating from Step 3 for the unrated player is less than 100, then change the rating to 100.

Step 4: For every player, calculate an intermediate rating with the appropriate rating formula.

- If a player's rating R_0 from Step 1 is based upon 8 or fewer games ($N \leq 8$), or if a player's game outcomes in all previous events have been either all wins or all losses, then use the "special" rating formula, with "prior" rating R_0.
- If a player's rating R_0 from Step 1 is based upon more than 8 games ($N > 8$), and has not been either all wins or all losses, use the "standard" rating formula (see Section 2.3). Note that the standard formula is used even if the "effective" number of games from Step 2 is less than or equal to 8.

In the calculations, use the opponents' pre-event ratings in the computation (for players with pre-event ratings). For unrated opponents who are assigned $N = 0$ in Step 1, use the results of Step 3 for their ratings. For unrated opponents who are assigned $N > 0$ in Step 1, use their assigned rating from Step 1.

If the resulting rating from Step 4 is less than 100, then change the rating to 100.

Step 5: Repeat the calculations from Step 4 for every player, again using a player's pre-event rating (or the assigned ratings from Step 1 for unrated players) to perform the calculation, but using the results of Step 4 for the opponents' ratings. If the resulting rating from Step 5 is less than 100, then change the rating to 100.

These five steps result in the new set of post-event ratings for all players.

2 Details of the Rating Algorithm

This section describes the details of the rating algorithm, including the computation for the "effective" number of games, the "special" rating formulas, and the "standard" rating formulas.

2.1 Effective number of games

For each player, let N be the number of tournament games the player has competed, or, for unrated players, the value assigned from Step 1 of the algorithm. Let R_0 be the player's pre-event rating, or, for unrated players, the imputed rating assigned from Step 1. Let

$$N^* = \begin{cases} 50/\sqrt{1 + (2200 - R_0)^2/100000} & \text{if } R_0 \leq 2200 \\ 50 & \text{if } R_0 > 2200 \end{cases} \quad (1)$$

Define the "effective" number of games, N', to be the smaller of N and N^*. As a result of the formula, N' can be no larger than 50, and it will usually be less, especially for players who have not competed in many tournament games. Note that N' is a temporary variable in the computation and is not saved after an event is rated.

Example: Suppose a player's pre-event rating is $R_0 = 1700$ based on $N = 30$ games. Then according to the formula above,

$$N^* = 50/\sqrt{1 + (2200 - 1700)^2/100000} = 50/\sqrt{3.5} = 26.7$$

Consequently, the value of N' is the smaller of $N = 30$ and $N^* = 26.7$, which is therefore $N' = 26.7$. So the effective number of games for the player in this example is $N' = 26.7$.

2.2 Special rating formula

This procedure is to be used for players with either $N \leq 8$, or players who have had either all wins or all losses in all previous rated games.

The algorithm described here extends the old provisional rating formula by ensuring that a rating does not decrease from wins or increase from losses. In effect, the algorithm finds the rating at which the attained score for the player equals the sum of expected scores, with expected scores following the "provisional winning expectancy" formula below. For most situations, the resulting rating will be identical to the old provisional rating formula. Instances that will result in different ratings are when certain opponents have ratings that are far from the player's initial rating. The computation to determine the "special" rating is iterative, and is implemented via a linear programming algorithm.

Define the "provisional winning expectancy," PWe, between a player rated R and his/her

i-th opponent rated R_i to be

$$\text{PWe}(R, R_i) = \begin{cases} 0 & \text{if } R \leq R_i - 400 \\ 0.5 + (R - R_i)/800 & \text{if } R_i - 400 < R < R_i + 400 \\ 1 & \text{if } R \geq R_i + 400 \end{cases}$$

Let R_0 be the "prior" rating of a player (either the pre-event rating for rated players, or the Step 1 imputed rating for unrated players), and N' be the effective number of games. Also let m be the number of games in the current event, and let S be the total score out of the m games (counting each win as 1, each loss as 0, and each draw as 0.5).

The variables R_0' and S', which are the adjusted initial rating and the adjusted score, respectively, are used in the special rating procedure. If a player has competed previously, and all the player's games were wins, then let

$$\begin{aligned} R_0' &= R_0 - 400 \\ S' &= S + N' \end{aligned}$$

If a player has competed previously, and all the player's games were losses, then let

$$\begin{aligned} R_0' &= R_0 + 400 \\ S' &= S \end{aligned}$$

Otherwise, let

$$\begin{aligned} R_0' &= R_0 \\ S' &= S + \frac{N'}{2} \end{aligned}$$

The objective function

$$f(R) = N' \times \text{PWe}(R, R_0') + \left(\sum_{i=1}^{m} \text{PWe}(R, R_i) \right) - S'$$

which is the difference between the sum of provisional winning expectancies and the actual attained score when a player is rated R, is equal to 0 at the appropriate rating. The goal, then, is to determine the value of R such that $f(R) = 0$ within reasonable tolerance. The procedure to find R is iterative, and is described as follows.

Let $\varepsilon = 10^{-7}$ be a tolerance to detect values different from zero. Also, let $x_0 = R_0' - 400$, $y_0 = R_0' + 400$, and, for $i = 1, \ldots, m$, $x_i = R_i - 400$, $y_i = R_i + 400$. Denote the unique x_i and y_i, $i = 0, \ldots, m$, as the collection

$$S_z = \{z_1, z_2, \ldots, z_Q\}$$

If there are no duplicates, then $Q = 2m + 2$. These Q values are the "knots" of the function f (essentially the value where the function "bends" abruptly).

1. Calculate

$$M = \frac{N'R_0' + \sum_{i=1}^{m} R_i + 400(2S - m)}{N' + m}$$

   This is the first estimate of the special rating (in the actual implemented rating program, M is set to R_0', but the final result will be the same – the current description results in a slightly more efficient algorithm).

2. If $f(M) > \varepsilon$, then

   (a) Let z_a be the largest value in S_z for which $M > z_a$.

   (b) If $|f(M) - f(z_a)| < \epsilon$, then set $M \leftarrow z_a$. Otherwise, calculate

$$M^* = M - f(M)\left(\frac{M - z_a}{f(M) - f(z_a)}\right)$$

   - If $M^* < z_a$, then set $M \leftarrow z_a$, and go back to 2.
   - If $z_a \leq M^* < M$, then set $M \leftarrow M^*$, and go back to 2.

3. If $f(M) < -\varepsilon$, then

   (a) Let z_b be the smallest value in S_z for which $M < z_b$.

   (b) If $|f(z_b) - f(M)| < \epsilon$, then set $M \leftarrow z_b$. Otherwise, calculate

$$M^* = M - f(M)\left(\frac{z_b - M}{f(z_b) - f(M)}\right)$$

   - If $M^* > z_b$, then set $M \leftarrow z_b$, and go back to 3.
   - If $M < M^* \leq z_b$, then set $M \leftarrow M^*$, and go back to 3.

4. If $|f(M)| \leq \varepsilon$, then let p be the number of i, $i = 1, \ldots, m$ for which

$$|M - R_i| \leq 400.$$

   Additionally, if $|M - R_0'| \leq 400$, set $p \leftarrow p + 1$.

(a) If $p > 0$, then exit.

(b) If $p = 0$, then let z_a be the largest value in S_z and z_b be the smallest value in S_z for which $z_a < M < z_b$. If

- $z_a \le R_0 \le z_b$, then set $M \leftarrow R_0$.
- $R_0 < z_a$, then set $M \leftarrow z_a$.
- $R_0 > z_b$, then set $M \leftarrow z_b$.

If the final value of M is greater than 2700, the value is changed to 2700. The resulting value of M is the rating produced by the "special" rating algorithm.

2.3 Standard rating formula

This algorithm is to be used for players with $N > 8$ who have not had either all wins or all losses in every previous rated game.

Define the "Standard winning expectancy," We, between a player rated R and his/her i-th opponent rated R_i to be

$$\mathrm{We}(R, R_i) = \frac{1}{1 + 10^{-(R-R_i)/400}}$$

The value of K, which used to take on the values 32, 24 or 16, depending only on a player's pre-event rating, is now defined as

$$K = \frac{800}{N' + m},$$

where N' is the effective number of games, and m is the number of games the player completed in the event. The following are example values of K for particular values of N' and m.

N'	m	Value of K
6	4	80
6	6	66.67
6	10	50
20	4	33.33
20	6	30.77
20	10	26.67
50	4	14.81
50	6	14.29
50	10	13.33

If $m < 3$, or if the player competes against any opponent more than twice, the "standard" rating formula that results in R_s is given by

$$R_s = R_0 + K(S - E)$$

where the player scores a total of S points (1 for each win, 0 for each loss, and 0.5 for each draw), and where the total winning expectancy $E = \sum_{i=1}^{m} \mathrm{We}(R_0, R_i)$.

If both $m \geq 3$ and the player competes against no player more than twice, then the "standard" rating formula that results in R_s is given by

$$R_s = R_0 + K(S - E) + \max(0,\ K(S - E) - B\sqrt{m'})$$

where $m' = \max(m, 4)$ (3-round events are treated as 4-round events when computing this extra term), and B is the bonus multiplier (currently B is set to 6 as of June 2008). The quantity

$$\max(0,\ K(S - E) - B\sqrt{m'})$$

is, in effect, a bonus amount for a player who performs unusually better than expected.

The resulting value of R_s is the rating produced by the "standard" rating algorithm.

2.4 Rating floors

The absolute rating floor for all ratings is 100. No rating can be lower than the absolute rating floor. An individual's personal absolute rating floor is calculated as

$$\mathrm{AF} = \min(100 + 4N_W + 2N_D + N_R,\ 150)$$

where AF is the player's absolute floor, N_W is the number of rated games won, N_D is the number of rated games drawn, and N_R is the number of events in which the player completed three rated games. The formula above specifies that a player's absolute floor can never be higher than 150. As an example, if a player has earned 3 wins, 1 draw, and has competed in a total of 10 events of at least three ratable games, then the player's absolute floor is $\mathrm{AF} = 100 + 4(3) + 2(1) + 10 = 124$.

A player with an established rating has a rating floor possibly higher than the absolute floor. Higher rating floors exist at 1400, 1500, 1600, ..., 2100. A player's rating floor is calculated by subtracting 200 points from the highest attained established rating, and then using the floor just below. For example, if an established player's highest rating was 1941, then subtracting 200 yields 1741, and the floor just below is 1700. Thus the player's rating cannot go below 1700. If an established player's highest rating was 1588, then subtracting 200 yields 1388, and the next lowest floor is the player's absolute floor, which is this player's current floor.

A player who earns the original Life Master (OLM) title, which occurs when a player keeps an established rating above 2200 for 300 (not necessarily consecutive) rated games, will obtain a rating floor of 2200. Achievement of other USCF titles do not result in rating floors.

A player's rating floor can also change if he or she wins a large cash prize. If a player wins over \$2,000 in an under-2000 context, the rating floor is set at the first 100-point level (up to 2000) which would make the player no longer eligible for that section or prize. For example, if a player wins \$2,000 in an under-1800 section of a tournament, then the player's rating floor would be 1800. Floors based on cash prizes can be at any 100-point level, not just the ones above based on peak rating.

3 Miscellaneous details

The following is a list of miscellaneous details of the rating system.

- All games played in USCF-rated events are rated, including games decided by time-forfeit, games decided when a player fails to appear for resumption after an adjournment, and games played by contestants who subsequently withdraw or are not allowed to continue. Games in which one player makes no move are not rated.

- The rating calculations apply separately to the QC and regular chess rating systems. Other than the use of imputing initial ratings for unrated players, there is no formal connection between these two systems.

- After an event, each players' value of N is incremented by m, the number of games the player competed in the event.

- Individual matches are rated with the following restrictions:

    1. Both players involved must be rated, with the difference in ratings not to exceed 400 points (the latest published rating prior to the match will be used for calculating the rating difference).
    2. No player may go up or down more than 200 rating points within a three-year period solely as a result of Match play.

- Ratings are stored as integers. During the rating of an event, intermediate computations are done using floating point arithmetic. When a post-event rating is less than the pre-event rating, the rating is rounded down. Conversely, when a post-event rating is greater than the pre-event rating, the rating is rounded up. The practical effect is that a positive result earns at least one point per event, and a negative result loses at least one point per event.

- The USCF Executive Director may review the rating of any USCF member and make the appropriate adjustments, including but not limited to imposition of a rating "ceiling" (a level above which a player's rating may not rise), or to the creation of "money floors" (rating floors that are a result of winning large cash prizes).