1. (a) In class we have proved:

**Proposition 1:** The integral defined by

\[
\int d\mu f = \lim_{n \to \infty} \int d\mu f_n
\]  

is well-defined. That is, if \( f_n \) and \( \tilde{f}_n \) are simple function sequences converging to \( f \) monotonically, then the limits of the sequences on the right of (1) for \( f_n \) and then with \( f_n \) replaced by \( \tilde{f}_n \) are identical to each other.

Show that this theorem is also true if \( f = \infty \) on a set of positive measure.

(b) Show the same (again using the in-class definition (1) of the integral \( \int d\mu f \) using simple functions) for

**Proposition 2:** Given a measure space \((\Omega, \mathcal{F}, \mu)\) and a non-negative measurable function \( f \) on \( \Omega \),

\[
\int_{\Omega} d\mu f = S \equiv \sup \left\{ \sum_{i} \left( \inf_{\omega \in A_i} f(\omega) \right) \mu(A_i) \right\}
\]  

where the sup is over all partitions \( \{A_i\} \) of \( \Omega \) into measurable sets.

That is, show the above holds if \( f = \infty \) on a set of positive measure.

2. Billingsley, problem 13.5 (Alternatively, you can let \( f = \lim_{n \to \infty} f_n \) and \( g = \lim_{n \to \infty} g_n \) with \( f_n \), \( g_n \) simple functions, and use Theorem 13.4.)

3. Problem 13.6

4. Problem 13.7

5. Problem 16.2

6. Problem 16.8