## Spring 2017

## MA 242 Prof. Mark Kon

#### Location:

Room 259, MCS Building (111 Cummington St.) Tel. 353-9549 Email: mkon@bu.edu

## **Office Hours:**

Tuesday 11 am - 12:15 pm Thursday 5:15-6:30 pm ... and by appointment

**Scope:** This course will cover rigorously various aspects of linear algebra. This subject is universally applicable in physics, applied mathematics, economics, management, and engineering, and any subject in which linear theories are important.

Text: The text for this class is Lay's *Linear Algebra and its Applications*, Fifth Edition.

**Homework:** Homework will generally be assigned out of the text, with an assignment corresponding to each lecture. Problems for the lectures of a given week will be collected on Thursday of the following week.

Homework scores will contribute to 25% of the final grade. The homework will be an integral part of the course, and it will be difficult to perform well without it. In general, it is good practice to complete a portion of the assignment after the corresponding lecture. Note that answers for odd numbered problems are in the back of the book.

In general an answer without an explanation will be considered insufficient, unless the answer is obvious. Problems which do not show reasons or work will not be given full credit. Please staple your homework.

**Please note:** No late homeworks will be accepted, unless you have a medical excuse (generally this means a note from a clinic). No makeups for exams will be given, and exams will not be excused without a medical reason.

**Class attendance:** To start, I will give everyone in the class the first 10 points out of the 100 possible just for class attendance. These points toward your final grade will be the easiest; however, remember that they can make the difference between an A and a B (or even a C+) in the class. Class attendance is required because the class experience will provide an essential intuitive understanding complementing your work at home, and will be needed to appreciate and comprehend the material in the course. Attendance will be taken at the beginning of class, so you will not get credit for attendance if you are late for class.

**Grading:** There will be two in-class exams and a final. The two exams will count for 15% and 20%, and the final, 30% of the final grade. Homework will determine 25% of the grade

and class attendance will determine 10% of the grade. Exams will cover in-class and assigned text material. Note that this class is small enough that it will also be possible to recognize individual effort, interest, and participation in the material.

**In class:** Questions in the class are welcome, and brief discussions of questions on problems and course material will be held at the beginning of each class, in which questions will be encouraged.

**Homework writeups:** Homework and problem solutions should be written up clearly - communication skills will determine a large part of your mathematical success in life, and will also determine a large part of your success in this course. Please make it a habit to rewrite homework solutions if they are not entirely in satisfactory final form on first writeup. In arguments involving proofs, you should focus on sentence construction and clarity of arguments in final write-ups. Proofs can create notorious difficulties if they are not written clearly and succinctly.

**Consultation on homework:** You are permitted and encouraged to consult with other students on homework problems, but this should be done on a general level of finding the solution of a problem. The final writeup of a problem set must be done by each student individually.

**Lectures:** After the lecture and before the next one, you should (i) read the relevant part of the text (unless this has been done before the lecture to facilitate your understanding); (ii) rewrite your lecture notes, adding any remarks, calculations, examples, that are needed so that you understand the material.

**Web Site for the Text:** Please familiarize yourself with the web site for the textbook (http://www.laylinalgebra.com). There are useful materials there. Your textbook should have a password which you can use at this site.

**Extra credit work - Computer Algebra:** Some work in this course, especially that involving large matrices, can benefit from a computer algebra package. In this class, we will favor the program Matlab (short for *Matrix Laboratory*). There will be extra credit assignments given out in class based on linear algebra work using Matlab.

Matlab is available on the BU Linux Virtual Lab computer system, and can be accessed from the BU system off campus if you have a VPN connection. The student version can be downloaded from Amazon or the Matlab web site for about \$100, and for about \$50 when purchaed at the BU Barnes & Noble store.

**Cheating:** Boston University's policies on cheating and plagiarism are quite unbending. If a student cheats on an exam, it is the policy of the University that he or she be given a hearing, the results of which may range anywhere up to permanent expulsion from the university. I consider cheating to be a serious offense. It is unfair to other students who have to work hard on material in the class. In addition, since grades are decided on a curve, high scores which are achieved through cheating put other students at a disadvantage. I urge students who are

aware of cheating to inform me of it when it occurs. During an exam this can be done discreetly in the process of asking question about the exam, for example, or by passing me a note. I will respect the desire for anonymity and noninvolvement of any student who informs me about a cheating situation. Please refer to the CAS academic conduct code for further information on how the university deals with cheating.

**Withdrawal:** Rules at the University will not allow students to withdraw from a class after the 8th week of classes - please make a note of this.

### Some notes on proofs

This is a note that I have written to one of my graduating students who was going on to take MA 511, our department's first course in Real Analysis. Though it is directed at another class, it has some useful observations about how to write proofs in any class - it may be worth referring back to at some points if you find it helpful

Hi Maria, Real Analysis (MA 511) is a very challenging course, and it requires real attention to keep up with it! I hope you have allocated enough time first of all to the class. Since this is such a difficult course, I recommend to students that they devote a good number of 2-hour blocks of time per week (pre-scheduled!) to the class.

I think you are right to focus on the question of proofs. It is a challenge to adjust to the thinking patterns needed to be able to write good ones. I would say that your steps could look like this (with plenty of time devoted to each one!)

**1.** Know the goal: a good proof is as \*simple\* as possible (even though that doesn't always mean it's very simple). Each declaration follows easily from the previous ones (thus in a complicated proof you might need a lot of declarations).

(a) A good proof is one that you would be able to look at a month later and understand - if that's not the case, what clarifications would you need to fill in?

2. Know to recognize a good proof when you see it (as mentioned above)

**3.** When you are unsure what to do, are you able to restate what you are trying to prove in terms of the definitions of all the terms in the statement you are proving?

4. You should realize that there are two sides to a proof:

(a) What you 'know' must be true and you are trying to explain (i.e., your intuition as to what is going on)

(b) What you write down as a formalization of that knowledge

5. You need to develop both sides of this process: Getting an intuition or 'picture' of what is going on is essential. Formalization of that picture will also have to come later.
(a) Do you know how to get such a picture? If not, try to simplify the statement by considering the \*simplest\* version (say only on the real line) that you still consider challenging

(b) Can you do this simplest version? If you can then you may improve your 'picture' of what is going on, and also move toward seeing how the proof in a more general case is done.(c) One mathematician has said that the secret to solving a problem (or doing a proof) is to find the simplest (most specialized) version of the statement that you still can't solve (or prove), and first solve that. That is, first find a simple example of what you are trying to prove, and prove this.

**6.** The formalization of the 'picture' or 'simple intuition' you have developed is the next step. This requires experience, partly obtained by \*simplifying\* the problems you are looking at (as

mentioned above) to the simplest version you can't solve. Try to formalize your proofs in these simple situations.

(a) An additional part of the formalization process is to get a popular book on proofs. Such a book will contain proofs of theorems that are simple standard proofs (e.g. that  $\sqrt{2}$  is irrational or that there are an infinite number of primes). What you should pay attention to is how the intuitive picture is translated into a formal proof, with simple sentences that follow from each other simply. This is your goal.

(**b**) If the formal proofs in the book make sense but you still feel you don't have the 'intuition' that should have led to them, then practice working backwards from formal proofs to the intuition that leads to them.

**7.** The last point is important - when you read proofs in your textbook you see formal proofs - do you know how to form a picture of what the author was thinking?

(a) You have to realize that no one thinks the way that proofs are written. Every mathematician thinks like you or me, keeping an intuitive picture of what is going on, and a 'dictionary' (that keeps improving) of how to translate informal pictures into formal statements and proofs.

(b) Part of understanding how informal pictures and formal proofs are related is essentially to learn to look at the proofs in your book and work \*backwards\*: figure out the intuition behind them.

(c) I realize this may be hard, since some proofs look so formal that you can't see the picture behind them. But sometimes you will find proofs in your book that you can do this with. You should try to get better in the process of 'informalizing' existing formal proofs - the road between intuition and formalization is a two way street and you get better at one direction (formalization) by getting better at the other direction (informalization and ability to form pictures).

**8.** A good way to master the connection between 'formalization' and 'informal thinking' is to ask yourself whether you could do the following exercise. Given a proof in your book, can you translate the entire proof into English, so that you can describe what is 'really going on'? This is something to practice and get better at.

## MA 242 Prof. Kon

# Problem Set 1 Due Thurs. January 26

**Some notes on homework writeups:** Please direct your attention to the course syllabus and the remarks on the desired form of homework solutions. Communication is a skill that is as important in Mathematics as in many other areas - and in this class we will pay attention to it! Please make sure your answers are written up clearly, with complete sentences, and with logic intact.

If you are writing a longer argument or a proof, feel free to number the parts of your argument sequentially (in a reasonable way), so that the final argument might have, say, five parts (numbered 1 through 5) which divide up the argument into logically whole pieces. Such a practice (for longer arguments only) gets you into the habit of parsing your thinking and logic, and makes it possible for the reader to follow your thinking more easily.

Important tip: In order to gain practice in writing good proofs, it is important to *rewrite* your proof.

Ask me in class or individually if you have problems in understanding or developing proofs.

Starred problems are optional - they are useful for understanding, but will not be graded

	READING	PROBLEMS
Lecture 1 Thurs. 1/19	1.1, 1.2	1.1/1, 3, 8, 9, 14, 15, 30, 33 1.2/1, 2, 4, 9, 14, 23, 26, 28, 33*

#### Additional problem:

A furniture manufacturer makes chairs, coffee tables, and dining room tables. Each chair requires 10 minutes of sanding, 6 minutes of staining, and 12 minutes of varnishing. Each coffee table requires 12 minutes of sanding, 8 minutes of staining, and 12 minutes of varnishing. Each dining room table requires 15 minutes of sanding, 12 minutes of staining, and 18 minutes of varnishing. The sanding bench is available 16 hours per week, the staining bench 11 hours per week, and the varnishing bench 18 hours per week. How many (per week) of each type of furniture should be made so that the benches are fully utilized?