MA 717 **PROBLEM SET 1**
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Lecture 1

Problems labelled with a * are optional (note they are not necessarily more difficult)

From the syllabus: In a course like this it is important to emphasize rigorous exposition. This is reflected in the need for rigor and care in written problem solutions. I will emphasize the need for good written communication of ideas in the homework assignments, and the ability to formalize intuitive mathematical notions clearly. This will include the requirement of well written and thought-out solutions. Try to write your proofs clearly, and in short sentences each of which follows easily from the previous ones. No argument or solution can be considered 'too simple.' In particular communication is as important a part of a mathematics class as it is any other. Feel free to consult with me regarding this.

1. Reed and Simon, problem I.1

2. Reed and Simon, problem I.3

3*. Reed and Simon, problem I.5

4. For x, $y \in \mathbb{R}^3$, define $\rho(x,y) = \max_{1 \leq i \leq 3} |x_i - y_i|$. Prove that ρ is a metric.

5. An example of completeness: Show that the vector space P of all polynomials on [0,1] is not complete under the L^2 norm $\|f\| = \sqrt{\int f^2(x)\,dx}$.

6. Some metrics: Consider the following functions on X $\times$ X, where X = C[0,1] = continuous functions on [0,1].

$$\rho_1(f_1,f_2) = \sup_{0 \leq x \leq 1} |f_1(x) - f_2(x)|$$

$$\rho_2(f_1,f_2) = \int_0^1 |f_1(x) - f_2(x)|\,dx$$

$$\rho_3(f_1,f_2) = \left(\int_0^1 |f_1(x) - f_2(x)|^2\,dx\right)^{1/2}$$

Prove that all three are metrics. Would ρ_3 still be a metric if the exponent 1/2 were removed from its definition?