MA 717
M. Kon

PROBLEM SET 14
Due Wed. 5/2/18

Lectures 24, 25, 26

Note: This problem set includes material that we will cover during the coming week - thus some of the later problems should be completed after the material is presented in class. The problems can be turned in on Wednesday May 2 in my mailbox (Room MCS 142, left side) or at your option on Thursday May 3, by 5 pm.

1. Show how Theorem VII.2 follows from Theorem VII.3. Prove just parts (a) to (d) and the uniqueness of the map.

2*. Reed and Simon, Problem 12 in Chapter VII.

3. Give the proof of Theorem VII.4 in more detail.

4. Find the spectrum of the Fourier transform. Are there any eigenfunctions?

5. Show that if **A** is the algebra of all polynomials on a bounded interval on the real line, and if $E(f)$ is the integral of f with respect to any bounded Borel measure, then $(\mathbf{A}, E)$ is an integration algebra. Show also that if the interval is infinite this is in general not the case.

6. Let m be a gage on the W^* algebra **A** on the Hilbert space $\mathcal{H}$. Let **C** denote the set of all operators in **A** whose range is contained in that of a projection in **A** on which m is finite. Show that **C** is a 2-sided ideal in **A**.

7. **(a)** Prove that the Fourier transform is a unitary tranformation, using theorems we have stated or proved.
(b) Prove that the quantum Fourier transform is also unitary.