Suggestions - PS 7

7. Direct differences: Assume for a contradiction two different subspaces W_1 and $\widetilde{W}_1$ satisfy (1) for the same W_2. Assume a $\widetilde{w}_1 \in \widetilde{W}_1$ with $\widetilde{w}_1 \notin W_1$. If $v = \widetilde{w}_1$ then

$$v = \widetilde{w}_1 + 0.$$

and v is a sum of vectors in $\widetilde{W}_1$ and W_2. By (1) there must be $w_1^* \in W_1$ and $w_2^* \in W_2$ with

$$v = w_1^* + w_2^*.$$

Thus show

$$w_2^* = \widetilde{w}_1 - w_1^*.$$

Is the right side above orthogonal to the left side? What does that imply?

9. Characterization of orthogonal direct sums: Show that if $v = w_1 + w_2$ with $w_i \in W_i$ then the sum is unique. Hint: if there were two such other w_i', show $0 = (w_1'-w_1) + (w_2'-w_2)$, and take the inner product of both sides with a single vector.

10. Properties of the W_j: You can assume without loss $j' > j$. Then if $w_j \in W_j$ and $w_{j'} \in W_{j'}$, show $w_j \in V_{j+1}$. Why are w_j and $w_{j'}$ orthogonal? Are the spaces they are contained in orthogonal?

11. A basic inequality: Schwartz inequality for Hilbert spaces

14. Convolutions and the Fourier transform: Try some changes of variable.