Review of *Wavelets: An Analysis Tool*, by M. Holschneider.

For *Physics Today*, by Mark A. Kon

Wavelets have been steadily gaining strength as an analysis tool pure and applied mathematics, as well as disciplines which use mathematics, including physics, engineering, and neural network theory. The field arose almost full-force from observations made in the mid-eighties by Stephane Mallat and Yves Meyer that orthonormal bases for functions (of the same nature as familiar sine and cosine bases of Fourier analysis) could be obtained from essentially a single function ψ(x) through dilations and translations. The advantage of wavelets is the localization of these basis functions, since the dilations and contractions of the "mother wavelet" ψ(x) allow for functions in the basis which have arbitrarily small support (i.e., non-zero extent). Familiar Fourier analysis techniques on the entire real line use sines and cosines (or equivalently complex exponentials), which by their nature have support over the entire line, causing confusion when specific local features of functions are sought from the Fourier transform. For example, local singularities of the function being expanded will result in slow convergence of the Fourier transform everywhere. This is one of several problems alleviated by wavelets.

Wavelet theory is one of the few fields in mathematics which produces little tension between its theory and applications. There are few sacrifices in rigor that need to be made to present a full theory of wavelets, and it is possible to present applied results using wavelets (e.g., on processing of EEG data) without any sacrifice of rigor. This has attracted many pure mathematicians into the fold, including a large number of very talented harmonic analysts. France has traditionally been strong in harmonic analysis, and the fact that wavelets originated there has led to a large amount of very high level work (as well as associated collaboration and conferences) there. Books on the subject originating from there include that of Meyer (*Wavelets and Operators,* Cambridge, 1992; translation). Another more standard reference originating in the U.S. is the book of Daubechies (*Ten Lectures on Wavelets,* SIAM Press, 1992)

This book's author is a physicist at Marseille who has made a number of contributions (both theoretical and applied) in wavelet theory. In the true spirit of wavelet theory, this book covers both applied and theoretical aspects. It is intended as a text, and serves this purpose very well, aside from a lack of exercises (which would make the book much more marketable in American universities), and a somewhat terse exposition at some points. The book starts by providing numerous examples (complete with spectral diagrams) of wavelets used in continuous wavelet transforms, together with the theory needed to understand them. After a thorough exposition of the continuous theory, discrete wavelet expansions are explained. There is a novelty in the approach, since the spaces usually associated with such expansions are defined in terms of their sampling properties (i.e., what rate of sampling is necessary to identify a function in a given space). This should be of significant interest to workers in signal processing, an area in which wavelet theory is already a significant force.

Other related wavelet expansions are also considered, including non-orthogonal ones, and computational and reconstruction issues are considered. The occurrence of a Gibbs phenomenon in wavelet expansions is studied. Wavelet analysis of fractal functions such as Brownian motion sample paths and the nowhere differentiable Riemann-Weierstrass function are also presented. There are some theoretical extensions toward the end of the book, in the direction of multiresolution analysis on more general groups.

Being intended at least partially as a textbook, this book includes the fundamental theorems of analysis which are used, sometimes with proof. This can be a great advantage to a practicing physicist who wishes to obtain thumbnail sketches of the analysis theorems behind the results. The book also has the advantage of being relatively recent, which is important in a fast-moving field. All in all, the book can serve well in the role of a text, reference or tutorial in wavelets. It has a strong theoretical component and contains the foundations for most current applications of wavelet theory, and its novel approaches in some parts make it worthwhile also for an expert.

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