Finance Letters, 2004, **2 (6)**, 8-15

Framework for Analyzing Spatial Contagion between Financial Markets

Brendan O. Bradley [a] and Murad S. Taqqu [b, *]

[a] Acadian Asset Management Inc., USA
[b] Boston University, USA

Abstract

We present an alternative definition of contagion between financial markets, which is based on a measure of local correlation. We say that there is contagion from market X to market Y if there is more dependence between X and Y when X is doing badly than when X exhibits typical performance, that is, if there is more dependence at the loss tail distribution of X than at its center. The dependence is measured by the local correlation between X and Y. This yields a test for contagion, which does not require the specification of crisis and normal periods. As such, it avoids difficulties associated with testing for correlation breakdown, such as hand picking subsets of the data, and it provides a better understanding of the degree of dependence between financial markets.

Keywords: Contagion, Local Correlation, Correlation Breakdown, Crisis Period
JEL classification: C12, C14

1. INTRODUCTION

One commonly believes that international financial markets are *more* dependent during a crisis than they are during calmer periods. This extra dependence during times of crisis is often referred to as contagion between markets. If present, it may mitigate the benefits of diversification precisely when those benefits are needed most and have serious consequences for investors. Thus, gauging dependence between international financial markets is of great interest not only to financial theorists but to practitioners as well.

Proper diversification across financial markets requires an understanding of the dependence between them. This dependence is typically quantified through the correlation coefficient. Using correlation alone to describe dependence is problematic for asset allocation because if forces the investor to suppose that returns are normally distributed[1], involving only variances and covariances, or to use a quadratic utility function. If the investor is primarily concerned with catastrophic losses either one of these choices may be unattractive. This is because normally distributed returns do not assign sufficient probability mass to large losses and a quadratic utility measures risk through variance and therefore without regard to the tails of the distribution. A safety-first risk averse investor[2] concerned primarily with guarding against large losses would not use linear models and correlation as a measure of dependence. He or she would be concerned about a greater dependence in the tails rather than in the center of the multivariate distribution that governs market returns. This increased dependence can be due to (or defined as) contagion, that is that large losses in one market contribute to large losses in other markets.

* *Corresponding author*: Email: *murad@bu.edu* We would like to thank Ashis Gangopadhyay, Vladas Pipiras and Stilian Stoev for many valuable comments which led to an improved exposition of this material. This research was partially supported by the NSF Grants ANI-9805623 and DMS-0102410 at Boston University.

[1] More generally, elliptically distributed.

[2] Such an investor acts so as to minimize the probability of catastrophic losses (see Roy 1952; Levy and Sarnat, 1972; Arzac and Bawa, 1977; and Jansen, Koedijk and de Vries, 2000).

ISSN 1740-6242

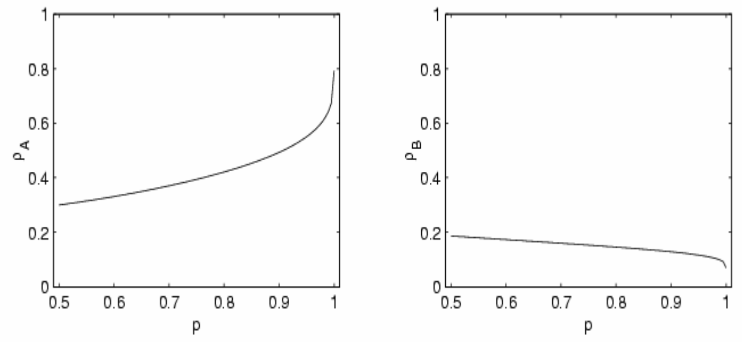

Figure 1. The conditional correlation of a standard bivariate normal distribution with unconditional correlation $\rho = 0.3$

Notes: The figure illustrates the sensitivity of conditional correlation to the choice of conditioning event. We display tail conditioning events $A = \{|X| > F_X^{-1}(p)\}$ and $B = \{X > F_X^{-1}(p)\}$. The situation described in the text, where volatility is large, corresponds to the two-sided case where one conditions on event A. In the one sided case, where one conditions on B, the conditional mean increases and the conditional variance decreases, leading to a decrease in ρ_B.

There is no universally accepted definition of contagion in the financial literature. Typical definitions involve an increase in the cross-market linkages after a market *shock*. See Pericoli and Sbracia (2001) or Forbes and Rigobon (2002), for example. The linkage between markets is usually measured by a conditional correlation coefficient, and the conditioning event involves a short *post-shock* or *crisis* time period. Contagion is said to have occurred if there is a significant increase in the correlation coefficient during the crisis period. This phenomenon is also referred to as *correlation breakdown*. Statistically, correlation breakdown corresponds to a change in structure of the underlying probability distribution governing the behavior of the return series. Most tests for contagion attempt to test for such a change in structure, but these tests may be problematic. One difficulty was pointed out by Boyer, Gibson and Loretan (1999) who showed that the choice of conditioning event may lead to spurious conclusions. The conditioning event is usually taken to be a short time period following a crisis event and may lead one to believe that there is a change of structure, that is, an increase in the unconditional correlation during the crisis period, even when there is none. For example, suppose

$$(X, Y) \sim \mathcal{N}(\mu_X, \mu_Y, \sigma^2_X, \sigma^2_Y, \rho)$$

and suppose A is any event such that $0 < P_X(A) < 1$. A simple calculation shows that

$$\rho_A = \rho(X, Y \mid X \in A) = \rho \left(\rho^2 + (1-\rho^2) \frac{Var(X)}{Var(X \mid X \in A)} \right)^{-1/2} \tag{1}$$

This implies that $|\rho_A| > |\rho|$ whenever $Var(X \mid X \in A) > Var(X)$. These variances are estimated using their corresponding sample variances. If the event is chosen by looking where the data are more variable, one automatically ensures that $Var(X \mid X \in A) > Var(X)$ and hence $|\rho_A| > |\rho|$ even though there has been no change of structure. This is called *heteroscedasticity bias*. Such a bias exists in a typical test for contagion, where one identifies a crisis event in a given market and defines a short crisis period (often by looking at the data). This crisis period determines the conditioning event A in equation (1). In such a test, the (squared) volatility, $Var(X \mid X \in A)$, of market X during the short crisis period is typically higher than the normal (unconditional) volatility $Var(X)$. (These volatilities are estimated by the sample variance of the (daily) returns evaluated on the time interval under consideration.) Therefore one might mistakenly believe that a change of structure of the distribution of (X, Y) has taken place when in fact this is merely an artifact of the choice of conditioning event.

Figure 1 illustrates how the choice of conditioning event affects the conditional correlation. It traces out the conditional correlation for a pair of standard bivariate normal random variables X and Y with unconditional correlation $\rho = 0.3$. The conditioning events are chosen to be $A = \{|X| > F_X^{-1}(p)\}$ and $B = \{X > F_X^{-1}(p)\}$. In a financial context the event A would correspond to conditioning on high volatility periods, typical of the conditioning event of standard contagion tests. Event B corresponds to conditioning on large losses. Under the assumption of normally distributed returns, large losses are asymptotically independent, i.e., $\rho_B \to 0$ as $p \to 1$, while the conditional correlation ρ_A increases if one chooses the conditioning event as A, which is associated with an increase in volatility. Neither of the two graphs in Figure 1 provides evidence of a change of structure of the underlying distribution since, by construction, the unconditional distribution has constant correlation of $\rho = 0.3$.

Forbes and Rigobon (2002) also recognize the difficulty in testing for contagion in this way. They propose a solution in which they test a *bias corrected* version of the correlation coefficient. Their *corrected version*, which is supposed to account for the relative increase in volatility of market X during the crisis period relative to the normal (unconditional) volatility, is given by

$$\tilde{\rho} = \rho_A \left[1 + \delta(1 - \rho^2_A)\right]^{-1/2} \tag{2}$$

where $\delta = \mathrm{Var}(X \mid X \in A) \,/\, \mathrm{Var}(X) - 1$. The correction factor δ represents the relative increase in the volatility of market X during the crisis period relative to normal conditions. Equation (2) is found by simply substituting δ into equation (1), solving for the unconditional correlation ρ, and denoting it $\tilde{\rho}$.

This corrected version of the correlation coefficient, called a heteroscedasticity corrected correlation, is viewed as the *unconditional* correlation of markets X and Y *during the crisis period*. The authors then test for contagion as follows:

$$H_0:\ \tilde{\rho} \le \rho \qquad \text{(no contagion)}$$

$$H_1:\ \tilde{\rho} > \rho \qquad \text{(contagion)}$$

Using this test, contagion is rarely observed during the 1987 US Stock Market Crash, the 1994 Mexican Peso Crisis and the 1997 East Asian Crisis.

The difficulty with the Forbes and Rigobon procedure is that the power of the test[3] is very low due to the short crisis period. It is very difficult to detect a change of structure (contagion) even when present. To see why this is so, suppose ρ changes from $\rho_1 = \rho$ to $\rho_2 = \tilde{\rho} > \rho$, that is, there is indeed contagion. Estimate ρ_i, by the sample correlation r_i, $i = 1, 2$; but note that ρ_1 is estimated using n_1 large, and ρ_2 is estimated using n_2 small. Forbes and Rigobon use a stable period of about $n_1 \approx 466$ days and a crisis period of about $n_2 \approx 35$ days. The probability distribution of the sample correlation estimator r_i is difficult to obtain so Dungey and Zhumabekova (2001) propose to apply a Fisher transformation (see Kendall, Stuart and Ord, 1987, Section 16.32) in order to make the distribution of r_i closer to normal. The distribution of the estimator $\hat{\rho}_i = (1/2) \log [(1 + r_i) \,/\, (1 - r_i)]$ of ρ_i is approximately normally distributed[4] with mean $\overline{\rho}_i = (1/2) \log [(1 + \rho_i) \,/\, (1 - \rho_i)]$ and variance $s^2_i = 1 \,/\, (n_i - 3)$. Assuming independence, the corresponding normal test is

$$Z = (\hat{\rho}_1 - \hat{\rho}_2) \,/\, (s^2_2 + s^2_1)^{1/2} \tag{3}$$

Notice the dependence of the test on the sample size n_2 of the crisis period. The small value of n_2 yields a large s^2_2 and makes it very difficult for the test to detect a difference in the estimated correlations. Dungey and Zhumabekova (2001) show through simulation that the test is unable to detect a change in structure of the underlying distribution due to the small sample size of the crisis period. They sample from two populations with statistically significantly different correlations in quantities similar to those used in the Forbes and Rigobon tests and find that the test in unable to reject the (incorrect) null hypothesis of equal correlations. When applying a longer sample period for the crisis to the Forbes and Rigobon data, which yields a smaller volatility for the market during the crisis, Dungey and Zhumabekova find more evidence of contagion. The authors conclude that correlation tests of this sort are an unattractive approach to testing for contagion.

We propose a different approach. At a more intuitive level, contagion may be defined as an increase in the probability of a crisis in one market, given a crisis in another market.[5] This corresponds to some sort of

---

[3] The power of a test is its ability to reject the null hypothesis (no contagion) when it if false. It is the probability of not making a *type two* error.

[4] This assumes ρ_i is small and n_i moderately large, say $n_i > 50$. These assumptions may not always be true in practice.

[5] This is the first of five representative definitions of contagion in the financial literature from Pericoli and Sbracia (2001).

increased dependence in the loss tails of the joint probability distribution of returns of financial markets compared to the center of the distribution. Such increased dependence may always have been present, but because tail events are by definition rare, they have often been ignored and only the center of the distribution has been modeled.

Only recently has attention been paid to the modeling of tail of the joint distribution of financial markets. Two recent papers in this area are Hartmann, Straetmans and de Vries (2001) and Longin and Solnik (2001). As we do here, the authors concentrate on measures of dependence relevant to specific parts of the underlying probability distribution. Tools from Extreme Value Theory are used to quantify dependence in the tails of the distribution. In each paper, the distribution of (X, Y) is assumed to be in the domain of attraction of a multivariate extreme value distribution. Hartmann et al. (2001) use a nonparametric measure called the *stable tail dependence function* and tools from univariate extreme value theory to calculate the expected number of crashes given that at least one crash has occurred. This statistic is calculated on equity and government bond returns for the G-5 markets to show evidence of cross-market contagion. Longin and Solnik (2001) use a parametric model from extreme value theory to describe the joint tail of the probability distribution. The dependence parameter of their parametric model is then related to the correlation of the model transformed marginals. The model is applied to monthly returns of the G-5 markets and leads the authors to reject the modeling of the loss tail by the multivariate normal distribution.

In this paper, we consider a definition of contagion based on a *local* correlation coefficient and provide a test for contagion. The local correlation coefficient has the look and feel of the traditional correlation coefficient but is a nonparametric measure designed to handle *nonlinear* forms of dependence. The measure of local correlation does not require the specification of a crisis and non-crisis period or the use of a heteroscedasticity correction as in (2). It was first proposed by Bjerve and Doksum (1993) as a means of characterizing the strength of association between a response variable Y and covariate X over a range of values of the covariate. The measure of local correlation has many appealing properties and we propose to use it to test for contagion. We recognize that this simple definition of contagion may leave many unsettled. After all, we avoid all discussion of transmission channels, speculative attacks, etc. Our goal is to find if contagion between markets is present using only the data. We do not introduce bias by choosing specific periods of time or subsets where volatility is large. We also show that it may not be possible to describe the dependence between financial markets using a single measure, correlation, based on what may be an over-simplified linear model.

This paper proceeds as follows. In Section 2 we motivate and define the measure of local correlation. We illustrate with an example its ability to characterize different degrees of association and we list several of its properties. Contagion, based on local correlation, is defined in Section 3 and a test for contagion is introduced. The estimation procedure is described in Bradley and Taqqu (2005b) and its application to financial markets can be found in Bradley and Taqqu (2005a). We use, in that article, twelve international equity market return series from Datastream and test for contagion between the US and a given international market. In addition, we show how local correlation and the correlation curve may be used to quantify the dependence between bond and equity markets. An understanding of this dependence is crucial to an investor deciding on how best to allocate his or her assets across the bond and equity markets. We make the software written in support of this work freely available and describe its use in the appendix of Bradley and Taqqu (2005a).

2. LOCAL CORRELATION

Correlation is an often misunderstood and therefore misused measure of dependence. Although ubiquitous in the finance literature, correlation is but one of many different possible measures of dependence. The two recent books of Joe (1997) and Drouet-Mari and Kotz (2001) detail many others. Correlation is a measure of *linear dependence*, that is, two random variables X and Y with finite second moment are perfectly correlated if and only if $Y = \alpha + \beta X$ almost surely. The correlation between two random variables X and Y determines their dependence structure when the distribution of (X, Y) is Gaussian, or more generally, elliptical.[6] See Embrechts et al. (2002) and Bradley and Taqqu (2003) for details. In cases where the distribution of (X, Y) is non-elliptical, correlation may be a misleading measure of dependence. This is why we introduce a test for contagion based not on correlation, but on local correlation. This will allow us to avoid many of the difficulties associated with the traditional approaches to gauging dependence between markets which are based on correlation.

The concept of local correlation was proposed in by Bjerve and Doksum (1993) and Doksum et al. (1994). Their goal was to extend the connection between regression slopes, strength of relationships and variance

[6] Elliptical distributions may be thought of as distributions whose constant density contours are ellipsoids in R^n.

explained by regression to nonlinear models. Their measure of strength of relationship, namely local correlation, is appropriate outside the world of normal distributions. We shall now define it. Suppose first that

$$(X, Y) \sim \mathcal{N}(\mu_X, \mu_Y, \sigma^2_X, \sigma^2_Y, \rho)$$

As is well known,

$$Y \mid X = x \sim \mathcal{N}\{ \mu_Y + \rho (\sigma_Y / \sigma_X)(x - \mu_X) ;\ \sigma^2_Y (1 - \rho^2) \} \tag{4}$$

The linear regression model is

$$Y = \alpha + \beta X + \sigma \varepsilon \tag{5}$$

where $\varepsilon \sim \mathcal{N}(0, 1)$ is independent of X. In this case

$$m(x) \overset{def}{=} \mathbb{E}(Y \mid X = x) = \alpha + \beta X$$

with regression slope $m'(x) = \beta$. It also follows that the regression slope β is $\rho\, \sigma_Y / \sigma_X$ and therefore

$$\rho = \beta\, (\sigma_X / \sigma_Y) \tag{6}$$

From linear regression theory, we may also write the variance σ^2_Y of Y as a sum of the variance explained by the regression, namely $\beta^2 \sigma^2_X$, and the residual (unexplained) variance σ^2, that is,

$$\sigma^2_Y = \beta^2 \sigma^2_X + \sigma^2 \tag{7}$$

and hence

$$\rho = \sigma_X \beta \ / \ (\sigma^2_X \beta^2 + \sigma^2)^{1/2} \tag{8}$$

Relation (8) shows that the correlation ρ is determined by σ_X, the regression slope β, and the regression residual variance σ^2. Assuming σ_X and the residual variance σ^2 constant, the correlation is an increasing function of the regression slope β. Assuming σ_X and the regression slope β constant, the correlation is a decreasing function of the residual variance σ^2.

Suppose that X and Y represent the returns in two different markets. Traditional tests for contagion assume σ^2 constant and test for a statistically significant change in the regression slope β during crisis and normal periods. We will allow here the regression slope β and residual variance σ^2 to vary as a function of the covariate X = x. This motivates the following definition of local correlation.

Definition 2.1: *Let X and Y be two random variables with finite variance. The local correlation between Y and X at X = x is given by*

$$\rho(x) = \sigma_X\, \beta(x) \ / \ [\, \sigma^2_X \beta^2(x) + \sigma^2(x)\,]^{1/2} \tag{9}$$

where σ_X denotes the standard deviation of X, $\beta(x) = m'(x)$ is the slope of the regression function $m(x) = \mathbb{E}(Y \mid X = x)$ and $\sigma^2(x) = Var(Y \mid X = x)$ is the nonparametric residual variance.

Note that this definition assumes that the distribution of the covariate X is continuous so that the slope $\beta(x) = m'(x)$ can be defined. Whereas the correlation was appropriate for the case in (5), the local correlation provides a measure of dependence for the more general model

$$Y = m(X) + \sigma(X)\, \varepsilon \tag{10}$$

where ε is mean zero, unit variance and is independent of X. Thus X affects Y in two ways: through the mean level m(X) and through the standard deviation $\sigma(X)$ associated with the noise ε. These may be different at different values of X = x. The local correlation $\rho(x)$ depends not only on the slope $\beta(x) = m'(x)$ but also on $\sigma(X)$, and for a given value of the slope $\beta(x)$, the larger $\sigma(X)$, the smaller $|\rho(x)|$.

The standard linear regression model (5) is, in fact, particularly restrictive because there are not many joint distributions consistent with it. That model implies a linear regression function $\mathbb{E}(Y \mid X = x) = \alpha + \beta x$ and a constant scedastic function $\mathrm{Var}(Y \mid X = x) = \sigma^2$. The bivariate normal distribution is one of the few joint distributions whose regression (conditional mean) and scedastic (conditional variance) functions satisfy these criteria. Not even the marginals of a bivariate Student's t distribution (which is an elliptical distribution) may be fit by this model.[7] Therefore, the generality of (10) is better suited to modeling the dependence between markets outside the world of normality.

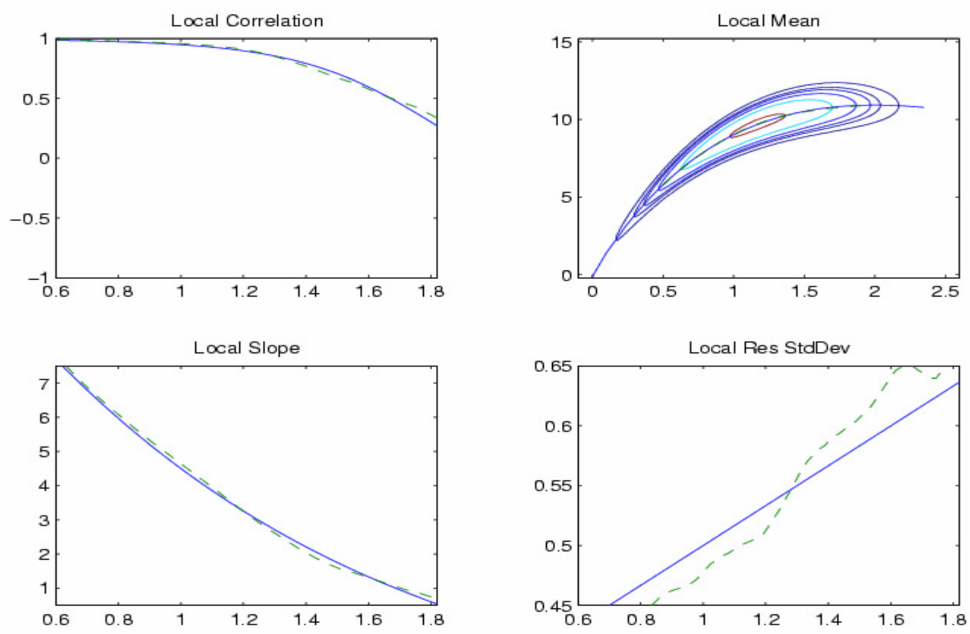

Figure 2. The correlation curve ρ(x), local mean m(x), slope β(x), and residual standard deviation σ(x) for the Twisted Pear Model

Notes: The true values are given by the solid lines. The dashed lines are the estimated values using the local polynomial modelling technique described in Bradley and Taqqu (2005b) based on a sample of size N = 2000. The upper right graph includes equi-density contours of the model.

Consider the following example from Doksum et al. (1994) known as the Twisted Pear Model. In equation (10), let $\varepsilon \sim \mathcal{N}(0; 1)$, (X, Y) have the joint density $f(x; y) = f(y \mid x)f(x)$ where $f(x)$ is $\mathcal{N}(1.2; (1/3)^2)$ and $f(y \mid x)$ is $\mathcal{N}(m(x); \sigma^2(x))$ with

$$m(x) = (x/10) \exp(5 - x / 2) \text{ and } \sigma^2(x) = ([1 + x / 2] / 3)^2$$

Contours of the joint density are given in Figure 2. The figure clearly shows two elements of the dependence between X and Y at work. The first is the change in the conditional mean $\beta(x) = m'(x)$ of Y as a function of the covariate X = x. The second is the variation about that mean, namely $\sigma(x)$, which clearly shows the heteroscedastic nature of Y as a function of covariate X. In this example, the local slope $\beta(x)$ is decreasing and the local residual variance $\sigma^2(x)$ is increasing. The local correlation measure captures both elements of the dependence, showing that the dependence starts out very strong for x near zero and decreases to zero as x approaches 2. It is simple to show that the local correlation satisfies many of the axioms that a global correlation measure is required to satisfy.

[7] The scedastic function for a bivariate Student's t_υ distribution, $\upsilon > 2$, is given by $\mathrm{Var}(Y \mid X = x) = (\upsilon\sigma^2 / \upsilon - 1)\{1 + (x - \mu_x)^2 / \upsilon\sigma^2_X\}$ where $\sigma^2 = \sigma^2_Y(1 - \rho^2)$, and so is a non-constant function of covariate x. (When $\upsilon \le 2$, the variances of X and Y are infinite). The regression function, $\mathbb{E}(Y \mid X = x)$, is linear and exactly the same as in the case where (X, Y) are normal. See, for example, Spanos (1999, p. 344).

Here we list several properties satisfied by local correlation. See Bjerve and Doksum (1993) for a more complete list.

Property 1: $-1 \le \rho(x) \le 1$

Property 2: $\rho(x)$ is invariant with respect to location and scale changes in X and Y.

Property 3: In the case of a true linear model, $\rho(x)$ reduces to the standard correlation measure.

Property 4: $\rho(x) = \rho$ for all bivariate normal distributions.

Property 5: X and Y independent implies $\rho(x) = 0$ for all x.

Property 6: If Y = m(X) is a non-constant function of X then $\sigma(X) = 0$ in (10) and one expects perfect dependence between X and Y. Indeed, one gets $\rho(x) = \pm 1$ for all x, where the sign of $m'(x)$ determines the sign of $\rho(x)$. In contrast, the linear correlation analog requires the function m to be linear in X in order to get $\rho = \pm 1$.

3. A DEFINITION OF CONTAGION BASED ON LOCAL CORRELATION

Let X denote the return of market X and Y denote the return of market Y over some fixed time horizon. Figure 3 illustrates the cumulative distribution function $F_X(x) = P[X \le x\}$ of X. Since the local correlation $\rho(x)$ measures the strength of the dependence between X and Y at different points of the distribution of X, we us it to define (spatial) contagion.

Definition 3.1: *We say that there is contagion from market X to market Y if*

$$\rho(x_L) > \rho(x_M) \tag{11}$$

where $x_M = F_X^{-1}(0.5)$ is the median of the distribution of X and $x_L = F_X^{-1}(0.025)$ is a low quantile of the distribution.

Contagion is therefore present when there is more dependence in the loss tail of the distribution, as measured by local correlation, than there is in the center of the distribution. Notice that our definition does not require any definition of crisis and normal periods and is not temporal in nature.

Nevertheless, the notion of crisis lurks in the background because large negative losses typically occur in crisis situations. We are therefore comparing what happens in crisis situations with what happens in tranquil periods. The estimation procedure will involve interpolation and hence the behavior around x_M and x_L.

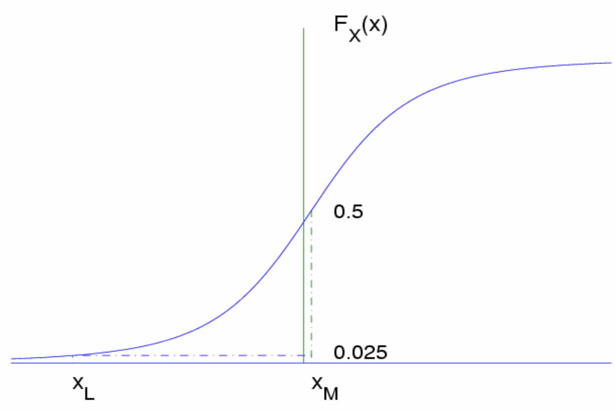

Figure 3: Distribution of a random variable X: [median x_M and low (2.5%) quantile $x_L = F_X^{-1}(0.025)$]

Note: x_M and x_L appear in the definition of contagion.

The choice of the 2.5% quantile can be modified in accordance with one's notions of crisis. In some cases the 2.5% quantile can be reached when the data is highly concentrated around the median. In this case, while contagion may have been found it may be irrelevant. This is why one must examine the data and see whether the losses incurred at this quantile are significant enough for one to care. Due to the heavy-tailed nature of international equity returns, the losses at this quantile are likely to be significant.

We can now define a test for contagion which checks for *more* dependence between X and Y when market X performs poorly than when market X has typical performance. Formally, we test

$$H_0: \ \rho(x_L) \le \rho(x_M) \qquad \text{(no contagion)}$$

$$H_1: \ \rho(x_L) > \rho(x_M) \qquad \text{(contagion)}$$

where x_L and x_M are as above.

The statistical implementation of this test is described on Bradley and Taqqu (2005b) and its application to financial markets can be found in Bradley and Taqqu (2005a). Using this test, we find evidence of contagion from the US equity markets to equity markets of several developed countries. We also find evidence of flight to quality from the US equity market to the US government bond market.

REFERENCES

Arzac, E. and V. Bawa (1977) Portfolio Choice and Equilibrium in Capital Markets with Safety-First Investors, *Journal of Financial Economics*, **4**, 277-288.

Bjerve, S. and K. Doksum (1993) Correlation Curves: Measures of Association as Functions of Covariate Values, *Annals of Statistics*, **21**, 890-902.

Boyer, B., M. Gibson and M. Loretan, M. (1999) Pitfalls in Tests for Changes in Correlations, *Technical Report*, Board of Governors of the Federal Reserve System, International Finance Discussion Paper.

Bradley, B. and M. Taqqu (2003) Financial Risk and Heavy Tails, in *Handbook of Heavy-Tailed Distributions in Finance*, Elsevier-Science, Amsterdam.

Bradley, B. and M. Taqqu (2005a) Empirical Evidence on Spatial Contagion between Financial Markets, *Finance Letters,* **3**, to appear.

Bradley, B. and M. Taqqu (2005b) How to Estimate Spatial Contagion between Financial Markets, *Finance Letters*, **3**, to appear.

Doksum, K., S. Blyth, E. Bradlow, X. Meng, and H. Zhao (1994) Correlation Curves as Local Measures of Variance Explained by Regression, *Journal of the American Statistical Association*, **89**, 571-582.

Drouet-Mari, D. and S. Kotz (2001) *Correlation and Dependence*. Imperial College Press.

Dungey, M. and D. Zhumabekova. (2001) Testing for Contagion using Correlations, *Pacific Basin Working Paper Series*, Economic Research Department, Federal Reserve Bank of San Francisco.

Embrechts, P., A. McNeil and D. Straumann (2002) Correlation and Dependence in Risk Management: Properties and Pitfalls, in M. Dempster and H. Moffatt, (eds.), *Risk Management: Value at Risk and Beyond*, Cambridge University Press. 176-223.

Forbes, K. and R. Rigobon, R. (2002) No Contagion, Only Interdependence: Measuring Stock Market Comovements, *Journal of Finance*, **57**, 2223-2261.

Hartmann, P., S. Straetmans. and C. de Vries (2001) Extreme Market Linkages in Crisis Periods, *Discussion Paper Series*, **2916**, Center for Economic Policy Research.

Jansen, D., K. Koedijk, K. and C. de Vries. (2000) Portfolio Selection with Limited Downside Risk, *Journal of Empirical Finance*, **7**, 247-269.

Joe, H. (1997) *Multivariate Models and Dependence Concepts*. Chapman and Hall.

Kendall, M., A. Stuart and J. Ord (1987) *Kendall's Anvanced Theory of Statistics*, **Vol. 1**, Oxford University Press.

Levy, H. and M. Sarnat (1972) Safety First - An Expected Utility Principle, *Journal of Financial and Quantitative Analysis*, **7**, 1829-1834.

Longin, F. andB. Solnik (2001)Extreme Correlation of International Equity Markets, *Journal of Finance*, **56**, 649-676.

Pericoli, M. and M. Sbracia (2001) A Primer on Contagion, *Working Paper*.

Roy, A. (1952) Safety First and the Holding of Assets, *Econometrica*, **20**, 431-449.

Spanos, A. (1999) *Probability Theory and Statistical Inference: Econometric Modeling with Observational Data*. Cambridge University Press.