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Empirical Evidence on Spatial Contagion Between Financial Markets

Brendan O. Bradley [a] and Murad S. Taqqu [b, *]

[a] Acadian Asset Management Inc., USA
[b] Boston University, USA

Abstract

We say that there is contagion from market *X* to market *Y* if there is more dependence between *X* and *Y* when *X* is doing badly than when *X* exhibits typical performance, that is, if there is more dependence at the loss tail distribution of *X* than at its center. This alternative definition of contagion between financial markets was introduced in Bradley and Taqqu (2004), where a test for contagion based on local correlation was presented. Using this test, we find evidence of contagion from the US equity markets to equity markets of several developed countries. We also find evidence of flight to quality from the US equity market to the US government bond market. We make the software written in support of this work freely available and describe its use in the appendix.

Keywords: Contagion, Local correlation, Correlation breakdown, Crisis period
JEL classification: C12, C14

1. INTRODUCTION

A test for contagion, based on local correlation, was introduced in Bradley and Taqqu (2004). The test checks for the presence of more dependence between market X and market Y when market X performs poorly than when market X has typical performance. We assume that the relationship between Y and X is of the form

$$Y = m(X) + \sigma(X)\varepsilon, \tag{1}$$

where ε is mean zero, unit variance and is independent of X. The local correlation between the random variables Y and X at $X = x$ equals

$$\rho(x) = \frac{\sigma_X \beta(x)}{\sqrt{\sigma_X^2 \beta^2(x) + \sigma^2(x)}} \tag{2}$$

where σ_X denotes the standard deviation of X, $\beta(x) = m'(x)$ is the slope of the regression function $m(x) = \mathbb{E}(Y \mid X = x)$ and $\sigma^2(x) = Var(Y \mid X = x)$ is the conditional variance. Contagion, using local correlation, is then defined as follows:

Definition 1.1. *We say that there is contagion from market X to market Y if*

$$\rho(x_L) > \rho(x_M) \tag{3}$$

where $x_M = F_X^{-1}(0.5)$ is the median of the distribution $F_X(x) = \mathbb{P}\{X \le x\}$ of X and $x_L = F_X^{-1}(0.025)$ is a low quantile of the distribution.

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Contagion is therefore present when there is more dependence in the loss tail of the distribution, as measured by local correlation, than there is in the center of the distribution. We use the following test for contagion:

$$H_0 \;:\; \rho(x_L) \le \rho(x_M) \;\; \text{(no contagion)}$$
$$H_1 \;:\; \rho(x_L) > \rho(x_M) \;\; \text{(contagion)}$$

where x_L and x_M are as above.

As indicated in Bradley and Taqqu (2004), the choice of the 2.5% quantile can be modified in accordance with one's notions of crisis. In some cases the 2.5% quantile can be reached when the data is highly concentrated around the median. In this case, while contagion may have been found it may be irrelevant. This is why one must examine the data and see whether the losses incurred at this quantile are significant enough for one to care. Due to the heavy-tailed nature of international equity returns, the losses at this quantile are likely to be significant. Figure 1 plots the US equity market returns for the time period considered here. The 2.5% quantile corresponds to a loss of about 2% of one's position in a single day.

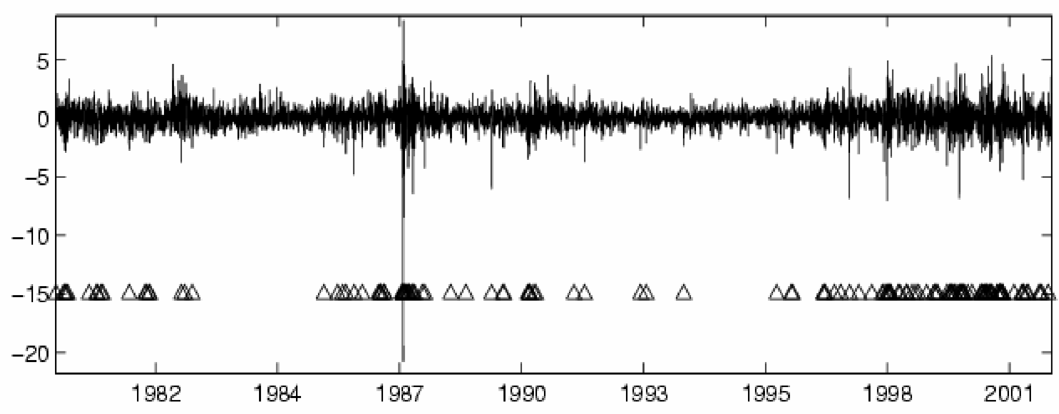

Figure 1. Time series plot of the US equity returns from from January 1980 to May 2002

All returns below the 2.5% quantile of the empirical distribution are indicated by triangles Δ.

We are now going to test for contagion between mature equity markets in the US and abroad and between the US government bond and equity markets using our methodology. When examining dependence between the US government bond and equity markets, we test for *flight to quality*. The estimation procedure for $\rho(x)$ is described in Bradley and Taqqu (2005). We make the software written in support of this work freely available and describe its use in the appendix.

2. DEVELOPED WORLD EQUITY MARKETS

We first consider developed world equity markets. We take the viewpoint of an investor with a consumption stream denominated in U.S. dollars and calculate returns based on U.S. dollars. We test for contagion between the U.S. and a given international market. The markets considered are Hong Kong, Japan, Australia, Belgium, Canada, France, Germany, Italy, Netherlands, Switzerland and the United Kingdom. The daily price index histories are calculated by Datastream, beginning in January 1980 and ending in May 2002. From the price indices, P_t, we construct the daily, weekly and monthly return indices. We experiment with both simple net returns, $100 \cdot (P_t - P_{t-1}) / P_{t-1}\%$, and continuously compounded (log) returns, $100 \cdot \log(P_t / P_{t-1})\%$. We find the results to be indifferent between the two and report results based on continuously compounded (log) returns. In order to account for the non-simultaneous market closings and for any other serial dependencies within and across markets, we also examine the residuals a_t of a two-dimensional vector autoregressive $VAR(p)$ model for $p = 1,\ldots,5$. The $VAR(p)$ model is given by

$$\Phi(B)r_t = (I - \Phi_1 B - \ldots - \Phi_p B^p)r_t = \phi_0 + a_t \tag{4}$$

where $r_t = [r_t^{US}, r_t^{Other}]^T$ and B is the back-shift operator, $Br_t = r_{t-1}$. The *VAR(p)* model removes serial linear dependencies within and across markets. Any concurrent relationship between markets is revealed by examination of the residuals a_t, $t = 1,\ldots,n$. This is done for the daily returns series only where the issue of non-simultaneous market closings may be relevant. In this case, $p = 1$ is the meaningful choice but we also consider values of p up to five to allow for linear dependence between and within markets for up to a week. Unless stated otherwise, the US market acts as the covariate market X.

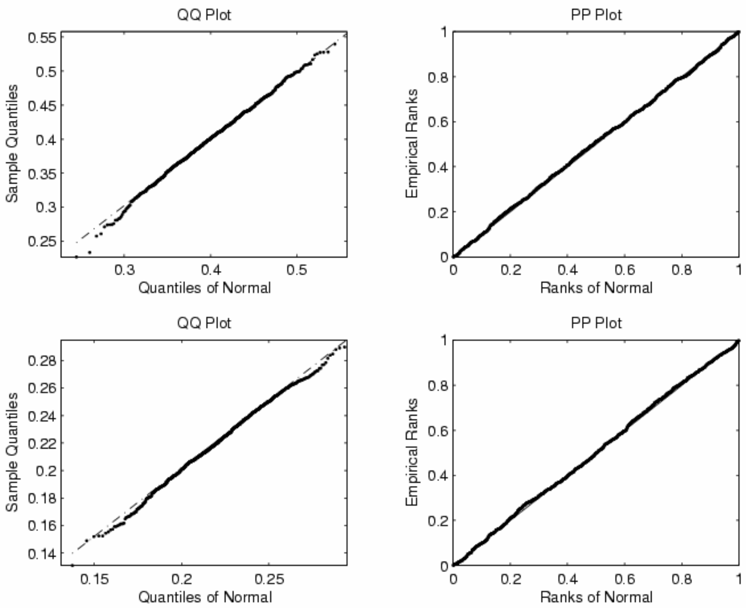

Figure 2. QQ and PP plots for the distribution of $\hat{\rho}(x_L)$ and $\hat{\rho}(x_M)$ versus the normal distribution obtained from 1000 Bootstrap samples

Clockwise from upper left: QQ plot for $\hat{\rho}(x_L)$, PP plot for $\hat{\rho}(x_L)$, QQ plot for $\hat{\rho}(x_M)$, PP plot for $\hat{\rho}(x_M)$.

In order to perform our test for contagion we need to make two assumptions about our estimators. First, we assume that the estimates $\hat{\rho}(x_L)$ and $\hat{\rho}(x_M)$ of $\rho(x_L)$ and $\rho(x_M)$ are independent. That is, we assume they are estimated from independent samples. We argue as follows. Assuming the sample $(X_i, Y_i), i = 1,\ldots,n$ are *i.i.d.* then the estimates $\hat{\rho}(x_L)$ and $\hat{\rho}(x_M)$ would be independent if the sets of data points used to compute them had no points in common. In other words, if no data point (X_i, Y_i) received positive weight in the calculation of both $\hat{\rho}(x_L)$ and $\hat{\rho}(x_M)$. Due to the nature of the local polynomial regression, this would obviously be the case if x_L and x_M were at least two bandwidths apart. Although this is not always the case, the points common to the estimation of local correlation of both x_L and x_M would be assigned very small weight and the problem should therefore be negligible. Second, we assume that the estimators $\hat{\rho}(x_M)$ and $\hat{\rho}(x_L)$ are normally distributed. The asymptotic normality of our estimator was established in Bradley and Taqqu (2005) under certain regularity conditions and asymptotic decay conditions on the bandwidths h_1 and h_2. In reality, the regularity conditions are difficult, if not impossible, to verify. Also, due to the finite samples of data, we have static bandwidths. The asymptotic theory gives the user confidence, but leaves one uncertain about how to proceed when faced with the reality of finite sample data. Therefore we rely on an examination of the distribution of the estimators obtained from 1000 Bootstrap samples of $\hat{\rho}(x_M)$ and $\hat{\rho}(x_L)$. Figure 2 shows Quantile-Quantile (QQ) and Probability-Probability (PP) plots of the Bootstrapped distribution of local correlation between US and French equity markets versus the normal distribution. Since the quantiles tend typically to bunch up in the center of the distribution and spread out in the tails, the QQ plots are used to check goodness of fit in the tails and the PP plots in the center of the distribution. The distributions of $\hat{\rho}(x_L)$ and $\hat{\rho}(x_M)$ appear to be well approximated by the normal distribution. The plots, in conjunction with Theorem 5.1 in Bradley and Taqqu (2005) give us enough confidence to proceed with our assumption of normality and construct a corresponding test statistic, namely, under the null hypothesis, the statistic

$$Z = \frac{\hat{\rho}(x_L) - \hat{\rho}(x_M)}{\sqrt{\hat{\sigma}^2_{\hat{\rho}(x_L)} + \hat{\sigma}^2_{\hat{\rho}(x_M)}}} \tag{5}$$

is standard normally distributed. We reject H_0 and conclude contagion between markets whenever $Z \geq z_{1-\alpha} = 1.65$ where $\alpha = 0.05$ and $z_{1-\alpha}$ is the $1-\alpha$ quantile of a standard normal distribution. $\hat{\rho}(x)$ and $\hat{\sigma}^2_{\hat{\rho}(x)}$ are computed respectively using equations (30) and (35) in Bradley and Taqqu (2005).

Results for daily returns are given in Table 1. The results are similar when we examine the residuals of the *VAR(p)* processes. For example, Table 1 shows seven instances of contagion between the markets tested and the US using daily returns. When examining the residuals of the *VAR(p)* processes we have $7,7,6,5,5$ instances of contagion when the lag p in (4) is $p = 1,2,3,4,5$ respectively. The seven instances in the *VAR(1)* and *VAR(2)* processes involve the same countries as those listed in Table 1 for the daily returns. As the lag p increases, some countries are dropped. When $p = 5$, the Netherlands and the UK are dropped from the list.

Table 1. Local correlations and contagion test statistic for daily returns in developed equity markets

Market	$\hat{\rho}(x_M)$	$\hat{\sigma}_{\hat{\rho}(x_M)}$	$\hat{\rho}(x_L)$	$\hat{\sigma}_{\hat{\rho}(x_L)}$	$\hat{Z}$	Contagion
Hong Kong	0.0872	0.0226	0.1106	0.0831	0.2724	NC
Japan	0.0948	0.0174	0.1150	0.0538	0.3569	NC
Australia	0.0590	0.0156	0.1239	0.0405	1.4949	NC
Belgium	0.1039	0.0175	0.2445	0.0506	2.6264	C
Canada	0.6436	0.0086	0.6197	0.0307	-0.7493	NC
France	0.2371	0.0148	0.3875	0.0359	3.8749	C
Germany	0.2181	0.0151	0.3625	0.0378	3.5474	C
Italy	0.1395	0.0157	0.2494	0.0421	2.4463	C
Netherlands	0.2677	0.0155	0.4042	0.0408	3.1297	C
Switzerland	0.1599	0.0185	0.3840	0.0502	4.1873	C
UK	0.3030	0.0152	0.4203	0.0408	2.6920	C

Contagion is defined as a stronger dependence, as measured by local correlation, in the (loss) tail of the distribution than in the center. Seven of the eleven tested markets exhibit contagion.

Our test shows contagion to be present, for daily returns, in seven of the eleven tested markets. Results for weekly and monthly returns are different. For weekly returns we find but a single instance of contagion between the Italian and US markets. For monthly returns we find contagion to be present in the German and Italian markets. It is not surprising that the results are dependent upon the time frequency. This is because investors react to new information differently depending on their investment time horizon. An investor planning for his or her retirement by allocating their portfolio across markets should care little about the daily disturbances of the markets. However banks and others required to meet daily minimum cash positions and margin calls need be very concerned about daily market conditions and may only care secondarily about longer horizons.

We also test for contagion when the US equity market acts as the dependent market. Here, we expect to see fewer instances of contagion since the US equity market is clearly the dominant world equity market[1]. In fact, Table 2 shows that we have have no instances of contagion when the US market acts as the dependent market. Lastly, we test for contagion between markets when using the daily returns series after filtering them for heteroscedasticity. That is, instead of using the return series $\{(X_i, Y_i),\ i = 1, \ldots, n\}$ in the test for contagion, we use the series $\{(\tilde{X}_i, \tilde{Y}_i),\ i = 1, \ldots, n\}$ where the tilde notation denotes innovation series after filtering for heteroscedasticity. We assume $X_t = \sigma_{X,t} \tilde{X}_t$ and model the volatility using a GARCH(1,1) model,

$$\sigma^2_{X,t} = \alpha_0 + \alpha_1 X^2_{t-1} + \beta_1 \sigma^2_{X,t-1}. \tag{7}$$

After filtering for heteroscedasticity, the resulting innovation series, $\tilde{X}_t$, are closer to being *i.i.d.* We find that the condition for stationarity ($\alpha_1 + \beta_1 < 1$) holds for all markets. For instance, for the US market, we get $\sigma^2_{X,t} = 0.0113 + 0.0671 X^2_{t-1} + 0.9231 \sigma^2_{X,t-1}$. We model the covariate market Y by $Y_t = \sigma_{Y,t} \tilde{Y}_t$ as well. Results for contagion tests using filtered returns are reported in Table 2. We see that when the US acts as the covariate market, we have 6 instances of contagion using the filtered series. When the US is the dependent market, we have none. Note that although filtering for heteroscedasticity effects the resulting test for contagion, we see that not all of the increased dependence between markets when the US market performs poorly can be attributed to dependent conditional volatility. Even after filtering for volatility, Table 2 reveals strong evidence of increased dependence between the markets when the US markets is performing poorly.

The case of Canada is interesting. The value of $\hat{Z}$ is very small and it remains relatively low even after filtering. The test, therefore, clearly indicates that there is no contagion. One would have thought perhaps that contagion would be present. But recall that contagion is viewed as an increase in dependence when things go wrong in the US market. Under regular circumstances, the dependence between the US and Canadian equity markets is already very strong. When the US market is performing very badly, the dependence between US and Canada may even decrease a little.

Table 2. Results of testimg for contagion between world equity markets

	US as covariate market X				US as dependent market Y			
	Unfiltered	Filtered	Unfiltered	Filtered	Unfiltered	Filtered	Unfiltered	Filtered
Market	$\hat{Z}$	C/NC	$\hat{Z}$	C/NC	$\hat{Z}$	C/NC	$\hat{Z}$	C/NC
Hong Kong	0.2724	NC	1.5140	NC	-1.6498	NC	-0.7459	NC
Japan	0.3569	NC	1.1176	NC	0.9723	NC	-0.2915	NC
Australia	1.4949	NC	2.1047	C	1.5773	NC	0.4266	NC
Belgium	2.6264	C	1.2792	NC	-0.2562	NC	-1.7102	NC
Canada	-0.7493	NC	1.0129	NC	-2.3656	NC	-2.2897	NC
France	3.8749	C	1.4242	NC	0.4681	NC	0.8236	NC
Germany	3.5474	C	1.8889	C	0.5092	NC	-1.7056	NC
Italy	2.4463	C	2.0681	C	-0.0556	NC	0.8551	NC
Netherlands	3.1297	C	2.1177	C	-0.9800	NC	-0.8838	NC
Switzerland	4.1873	C	2.2698	C	-0.4560	NC	-0.5230	NC
UK	2.6920	C	1.9387	C	-2.6038	NC	-2.8607	NC

We test using the Us equity market as both the covariate market X and the dependent market Y. We report results for daily returns and volatility filtered daily returns.

[1] As of May-2002, the US equity market had between 5 and 76 times the capitalization of the other markets we investigated.

3. BOND AND EQUITY MARKETS IN THE US

In addition to better understanding the dependence between international equity markets, for the purpose of asset allocation, it is crucial to understand the relationship between bond and equity markets. It is often believed that government bond markets do well when equity markets do not. This phenomenon is often referred to as a *flight to quality*, the idea being that as equity markets crash, investors hurry to the relative safety of the government bond market. Tests for flight to quality typically proceed in the same fashion as test for contagion, that is, crisis and non-crisis time periods are defined, and sample correlations are computed for each period respectively. In this case, however, instead of looking for an increase in the correlation between US bonds and stocks, flight to quality would manifest itself as a decrease in correlation. One might well expect this correlation to become negative, that is, as equities perform well below their average, government bond markets should perform well above their average.

Historically, the correlation of returns in the US Bond and Equity markets is small but positive. For example, the sample correlation between the US equity market and government bond market is about $r = 0.090$ for daily returns during the period from November 1986 to May 2002. Typical tests for flight to quality often show a negative correlation during a crisis period, presumably giving evidence of flight to quality. For example, Gulko (2000) tests for flight to quality in the US government bond and equity markets by identifying and aggregating over six crisis periods since 1970. The crisis periods are identified with crashes in the equity market, where a crash is defined by a loss of at least 5%. After identifying a crash, the crisis period is defined as a short period about the crash. Gulko defines a prologue period for each crash as the ten trading days preceding the crisis period. Then Gulko aggregates all the prologue periods and aggregates all the crisis periods. Fitting a separate simple linear regression model $Y = \alpha + \beta X + \varepsilon$ to the aggregated prologue and crisis periods, which contain 60 and 79 data points respectively, he finds the stock-bond correlation to be $+0.257$ in the prologue (pre-crisis) period and -0.445 in the crisis period. Tests of this nature suffer from the same sort of problems as their contagion counterparts because one is hand picking the crisis periods after the fact. Instead, we prefer to examine the issue of flight to quality in the context of the local correlation measure.

We consider returns from the US equity series used in the contagion tests and returns from a Merrill Lynch US Government Bond index of representative one to ten year maturity bonds[2]. The return series cover from November 1986 to May 2002. The covariate market X is the equity index and the dependent market Y is the bond index. Typical wisdom tells us that when equity markets do particularly well, bond markets do poorly and when equity markets do particularly poorly, bond markets do well. In terms of our local correlation measure this behavior should manifest itself by negative local correlation at both ends of the covariate's spectrum. Our test for flight to quality is similar to that for contagion except this time our test is:

$$\begin{aligned} H_0 &: \rho(x_L) \geq \rho(x_M) \text{ (no flight to quality)} \\ H_1 &: \rho(x_L) < \rho(x_M) \text{ (flight to quality)} \end{aligned}$$

where x_L and x_M are defined as in Definition 1.1.

Table 3. Local correlations and flight to quality test statistics for daily, weekly and monthly returns in the US government bond and equity markets from November 1986 to May 2002

Frequency	$\hat{\rho}(x_M)$	$\hat{\sigma}_{\hat{\rho}(x_M)}$	$\hat{\rho}(x_L)$	$\hat{\sigma}_{\hat{\rho}(x_L)}$	$\hat{Z}$	Flight to Quality (FTQ)
Daily	0.4275	0.0193	-0.2409	0.0811	-8.0147	FTQ
Weekly	0.3399	0.0480	-0.4096	0.1466	-4.8591	FTQ
Monthly	0.4424	0.1041	-0.5688	0.2348	-3.9372	FTQ

Flight to quality is defined as a weaker dependence, as measured by local correlation, in the (loss) tail of the equities distribution than in the center. We say there is a Flight to Quality (FTQ) if $\hat{Z} < -1.65$. the results indicate a very significant change from positive association in the center of the distribution to negative association in the tails of the distribution.

[2] Results do not change when looking at an *all maturity* government bond index.

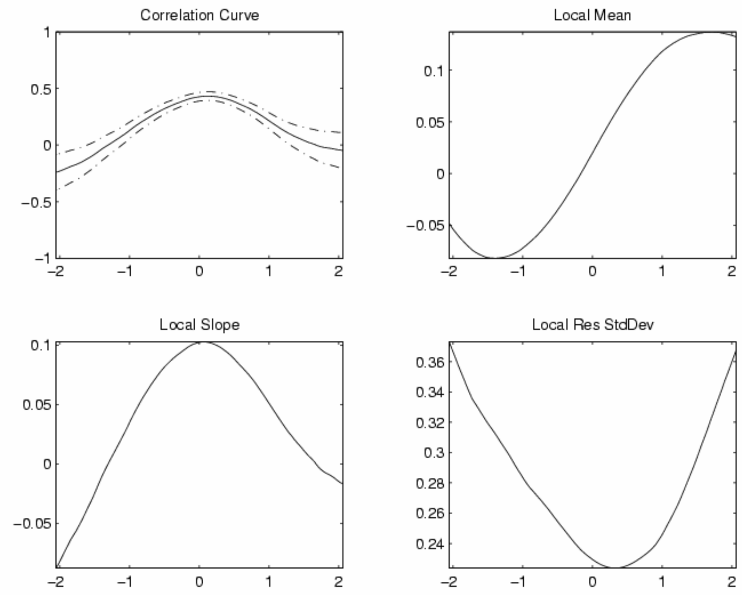

Figure 3. The correlation curve, local mean, slope and residual standard deviation for the US government bond market as a function of the (log) returns, expressed as a percent, of the US equity market

95% confidence intervals are attached using normality of the estimator and equation (35) in Bradley and Taqqu (2005).

Table 3 and Figure 3 confirm the expected behavior and gives credible evidence of flight to quality. The local correlation changes from significantly positive in the center of the distribution to significantly negative in the (equity loss) tail of the distribution. An investor relying only on the sample correlation r to quantify dependence between the equity and bond markets would come to the erroneous conclusion that the markets were nearly uncorrelated since $r = 0.09$. The local correlation measure shows that the markets have varying degrees of conditional dependence. On a typical day in the equity market, the association is strong and positive with a local correlation of $\hat{\rho}(x_M) = 0.43$. On a bad day in the equity market, the association is negative and fairly strong with a local correlation of $\hat{\rho}(x_L) = -0.24$. The use of an overall correlation between stocks and bonds, which gives $r = 0.09$, is much less informative because it can be viewed, roughly, as averaging over the whole range of values of X and thus is not adequate for judging flight to quality.

Similar behavior holds for weekly and monthly horizons and using heteroscedasticity filtered daily returns. Figure 3 is especially informative. It shows that the change in the dependence, as measured by local correlation, is a smooth function of the equity market returns. Although there is obviously a certain amount of smoothing involved in the local correlation modelling, a sharp change in the dependence between the equity and bond markets would still reveal itself on the correlation curve. We performed the following simulation to confirm that a sharp quantile based change of dependence would be evident on the correlation curve. Let $X \sim \mathcal{N}(0,\, 1^2)$. Generate n_s random variates $\{X_i, i = 1, \ldots, n_s\}$ with distribution F_X. Then generate Y_i conditionally on $X_i = x_i$ so that

$$
\begin{aligned}
Y_i \mid X_i = x_i \geq F_X^{-1}(0.075) = -1.44 &\sim \mathcal{N}\left(\rho_1 \frac{\sigma_1}{\sigma_X} x_i \,,\; \sigma_1^2 (1 - \rho_1^2) \right) = \mathcal{N}(0.3 x_i, (\sqrt{0.91})^2) \\
Y_i \mid X_i = x_i < F_X^{-1}(0.075) = -1.44 &\sim \mathcal{N}\left(\rho_2 \frac{\sigma_2}{\sigma_X} x_i \,,\; \sigma_2^2 (1 - \rho_2^2) \right) = \mathcal{N}(1.4 x_i, (\sqrt{2.04})^2),
\end{aligned} \tag{7}
$$

where $\sigma_1 = 1, \rho_1 = 0.3, \sigma_2 = 2$ and $\rho_2 = 0.7$. Figure 4 shows the correlation curve and its constituent parts for the simulation. Although partially smoothed, there is still a sharp shift in the local correlation clearly visible. The correlation curve for the equity and bond markets in Figure 3 reveals no such shift in dependence. Our results show that the dependence structure between equity and government bond markets is considerably more complicated than that which any simple linear model(s) can capture. Finally, we note that the results are not an artifact of the crash of 1987. They do not change when we exclude it: similar results were obtained looking at the series from January 1990 to May 2002.

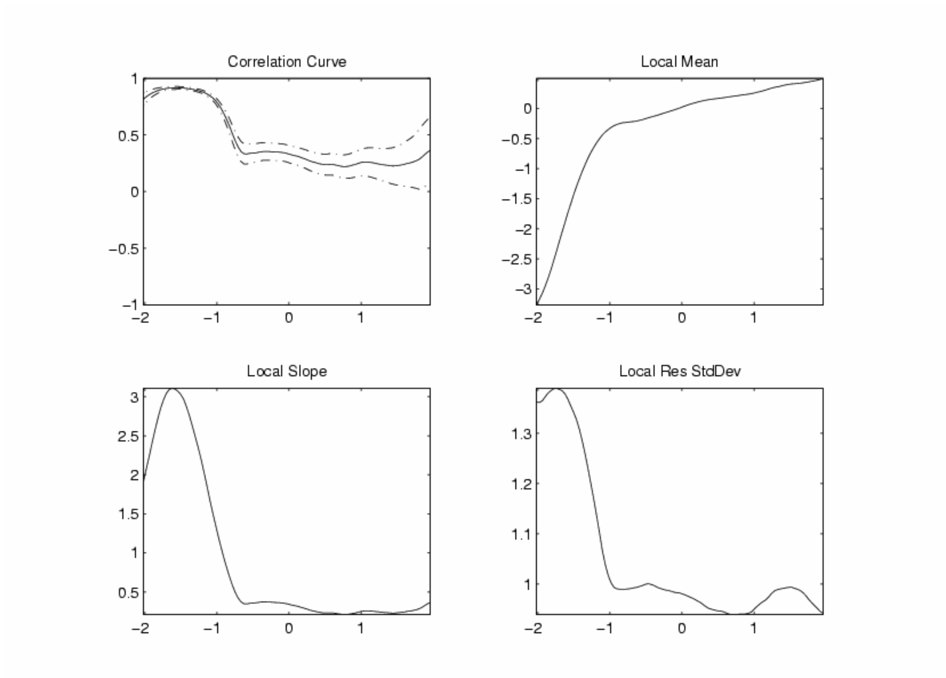

Figure 4. The correlation curve, local mean, slope and residual standard deviation of the simulated data $\{(X_i, Y_I), i = 1, \ldots, n_s\}$ where $n_s = 5000$, $X \sim N(0, 1^2)$ and Y_i is generated conditionally on $X_i = x_i$ according to equation (7)

4. CONCLUSION

Understanding dependence in the tails of the distribution of returns of financial markets is of great concern for those looking to diversify financial risk across international boundaries or across different asset classes. Issues such as contagion between international markets and flight to quality from equity to government bond markets can have a dramatic effect on how to best allocate one's assets. In this paper we define contagion and flight to quality based on a measure of local correlation. The measure is based on a general non-linear model which does not presuppose, either implicitly or explicitly, any specific probability law on the distribution of returns. Since it is not explicitly temporal, it avoids the hand picking of *crashes* and the defining of *crisis* periods, which we believe to be a serious impediment to most traditional tests for contagion and flight to quality. Instead, local correlation measures the dependence between Y and covariate X locally throughout the support of the distribution of X. This allows one to gain a better understanding of the relationship between markets in the tails of the distribution. This understanding is crucial for those concerned with guarding against catastrophic losses.

Our empirical study based on local correlation suggests both contagion and flight to quality to be prevalent. For daily returns, our tests for contagion with the US equity market reveal contagion in about half of the markets tested. Lower return frequencies revealed different amounts of contagion between markets. We found a single instance of contagion for weekly returns and two instances of contagion for monthly returns. A similar definition for flight to quality between US equity and government bond markets revealed significant evidence of flight to quality for daily and weekly returns. Additionally, the correlation curve revealed a surprising relationship for the dependence between (US) stocks and bonds. The association, as measured by local correlation, is quite high on a typical day in the equity market and smoothly decreases as equities perform particularly well or particularly poorly. The association is strongly negative at both ends of the equity return spectrum. The dramatic difference in the strength of dependence between equity and government bond markets from a typical day in the equity markets and a very bad day, as measured by local correlation, lend support to the popular theory of a quick, nearly instantaneous transition from positive to negative association during a crash in the equity market.

6. USING THE SOFTWARE

The software used to perform the contagion analysis in this paper was written in MATLAB and may be obtained from the authors. We illustrate its use in this section.

The MATLAB functions necessary to use the software are gathered in a directory called *ContagionDir*. The user should put his data in a MATLAB data file named, for example, *ReturnData.mat*. We assume here that the data file contains a *num_obsxnum_asset* MATLAB data array called *AssetReturns*. The array represents return data where *num_obs=the number of observations* and *num_asset=the number of assets* . Each row of this array corresponds to a joint observation of the *num_assets* assets. We will analyze a pair of assets at a time. A sample session using this software might proceed as follows.

Invoke MATLAB. Add the *ContagionDir* directory to MATLAB's working path. Here, we assume the directory is located in the *MyHomePath subdirectory*:

```
addpath('C:\MyHomePath\ContagionDir');
```

Load the data set into MATLAB workspace:

```
load('C:\MyHomePath\ReturnData');
```

The MATLAB workspace will now contain the array *AssetReturns*. Next, pick a pair of assets from the array. We pick i , $1 \le i \le num\_assets$ to be the covariate market X and j , $1 \le j \le num\_assets$, $j \ne i$ to be the dependent market Y . Choose $i = 1$ and $j = 2$ for example:

```
i = 1; j = 2;
X = AssetReturns(:,i); Y = AssetReturns(:,j);
```

Define a set of target points for which we would like local correlation estimates. For example, suppose we want 101 equally spaces estimates of the local correlation from $x_{\mathtt{min}} = F_X^{-1}(0.025)$ to $x_{\mathtt{max}} = F_X^{-1}(0.975)$. Then enter

```
num_targets = 101;
x_min = prctile(X, 2.5);
x_max = prctile(X, 97.5);
x_0 = linspace(x_min, x_max, num_targets)';
```

x_0 is now a column vector of target points. The following command estimates the local correlation at the target points x_0 and plots the correlation curve and its associated parts (see Figure 3):

```
plot_flag = 1;
[Rho, Beta, Sigma, StdRho] = CorrCurve(Y, X, x_0, plot_flag);
```

The function *CorrCurve* returns the following data.

- *Rho* (num_targets × 1) array of local correlation estimates $\hat{\rho}(x_0)$.
- *Beta* (num_targets × 3) array of local regression coefficients. The first column corresponds to local mean estimates $\hat{m}(x_0)$, the second corresponds to the local slope estimates $\hat{\beta}(x_0)$ and the third corresponds to $1/2!$ times the estimate of the second derivative of the regression function, $\hat{m}^{(2)}(x_0)/2!$, at the target points x_0 (see equation (8) in Bradley and Taqqu, 2005).
- *Sigma* (num_targets × 1) array of local residual standard deviation estimates $\hat{\sigma}(x_0)$.
- *StdRho* (num_targets × 1) array of local standard deviations of the estimator Rho, $\hat{\sigma}_{\hat{\rho}(x0)}$ (see equation (35), in Bradley and Taqqu, 2005) to be used in establishing confidence intervals.

To examine the data, type Rho, Beta, Sigma or StdRho at the MATLAB command prompt. To save the results to a MATLAB data file called *Results.mat* type:

```
save 'C:\MyHomePath\Results' Rho Beta Sigma StdRho
```

To examine QQ and PP plots of the distribution of $\hat{\rho}(L)$ and $\hat{\rho}(M)$ obtained from *num_boot* Bootstrap resamples, enter the following.

```
num_boot = 1000;
BootstrapLocalCorr(Y, X, num_boot);
```

The plots will be displayed automatically.

Tests for contagion then proceed by constructing the statistic $\hat{Z}$ as in (5). If x_0 has been constructed as above, then x_L (lower quantile) is the first element of the x_0 array and x_M (median point) is the 51^{st}. If we are testing for contagion at the $1-\alpha = 0.95$ confidence level then type:

```
confidence_level = 0.95;
lower_idx = 1;
median_idx = 51;
Contagion = TestContagion(Rho, StdRho, lower_idx, median_idx, confidence_level);
```

The scalar *Contagion* is either 1, if the null hypothesis is rejected (contagion), or 0 if the null hypothesis is not rejected (no contagion).

To perform the analysis on the residuals of the *VAR(p)* (see equation (4)) we instead call the function *CorrCurveVARp* instead of *CorrCurve*. The *VAR(p)* modeling uses functions from the econometrics toolbox written by James P. LeSage, Dept of Economics at the University of Toledo. It is freely available at http://www.spatial-econometrics.com/. Once downloaded, add the directory and subdirectories to MATLAB path as above. To estimate the local correlation on the residuals of a *VAR(p)*, $p = 2$ model type:

```
p_lag = 2;
[Rho, Beta, Sigma, StdRho] = CorrCurveVARp(Y, X, x_0, p_lag, plot_flag);
```

REFERENCES

Bradley, B. and M. Taqqu (2004) Framework for analyzing spatial contagion between financial markets, *Finance Letters,* **2 (6)**, 8-15.

Bradley, B. and M. Taqqu (2005) Empirical evidence on spatial contagion between financial markets, *Finance Letters,* **3 (1)**, 64-76.

Gulko, L. (2000) Decoupling, *Technical Report*, Paloma Partners, USA (lgulko@paloma.com).