MA 226 Worksheet
Second order equations (4) Key

Find the general solution to the following second order linear differential equations:

1.) $y'' + 5y' + \frac{75}{2}y = 2\cos\frac{5}{2}t$

1A.) $y = c_1 e^{\frac{-5}{2}t}\cos\frac{5\sqrt{5}}{2}t + c_2 e^{\frac{-5}{2}t}\sin\frac{5\sqrt{5}}{2}t + \frac{16}{725}\sin\frac{5}{2}t + \frac{8}{145}\cos\frac{5}{2}t$

2.) $y'' + 3y' + 12y = t^3 - 1$

2A.) $y = c_1 e^{\frac{-5}{2}t}\cos\frac{\sqrt{39}}{2}t + c_2 e^{\frac{-3}{2}t}\sin\frac{\sqrt{39}}{2}t + \frac{1}{12}t^3 - \frac{1}{16}t^2 - \frac{1}{96}t - \frac{9}{128}$

3.) $y'' + 4y' + 20y = \pi e^{-2t}\cos 4t$

3A.) $y = c_1 e^{-2t}\cos 4t + c_2 e^{-2t}\sin 4t + \frac{\pi}{8}te^{-2t}\sin 4t$

4.) $y'' + 16y = \sin 4t$

4A.) $y = c_1\cos 4t + c_2\sin 4t - \frac{1}{8}t\cos 4t$

5.) $y'' + 10y' + 2y = 4$

5A.) $y = c_1 e^{(-5-\sqrt{23})t} + c_2 e^{(-5+\sqrt{23})t} + 2$

6.) $y'' + 6y' + 5y = 3e^{-t}$

6A.) $y = c_1 e^{-t} + c_2 e^{-5t} + \frac{3}{4}te^{-t}$

7.) $y'' + 12y' + 3y = \sin 3t + \cos 3t$

7A.) $y = c_1 e^{(-6-\sqrt{33})t} + c_2 e^{(-6+\sqrt{33})t} - \frac{5}{222}\sin 3t + \frac{7}{222}\cos 4t$

8.) $3y'' + 6y' + 3y = -e^{-t}$

8A.) $y = c_1 e^{-t} + c_2 te^{-t} - \frac{1}{6}t^2 e^{-t}$

9.) $y'' + 16y' + 64y = t^2 + 1$

9A.) $y = c_1 e^{-8t} + c_2 te^{-8t} + \frac{1}{64}t^2 - \frac{1}{128}t + \frac{35}{2048}$

Find the solutions to the following initial value problems and graph their solutions:

10.) $y'' + 5y' + \frac{75}{2}y = 2\cos 2t$, $y(0) = 0$, $y'(0) = 1$.

10A.) $y = \frac{268}{4889}e^{\frac{-5}{2}t}\cos\frac{5\sqrt{5}}{2}t + \frac{8118}{4889\cdot 5\sqrt{5}}e^{\frac{-5}{2}t}\sin\frac{5\sqrt{5}}{2}t + \frac{268}{4889}\cos 2t + \frac{80}{4889}\sin 2t$

11.) $y'' + 3y' + 12y = 5$, $y(0) = 3$, $y'(0) = 1$.

11A.) $y = \frac{31}{12}e^{\frac{-3}{2}t}\cos\frac{\sqrt{39}}{2}t + \frac{\sqrt{39}}{4}e^{\frac{-3}{2}t}\sin\frac{\sqrt{39}}{2}t + \frac{5}{12}$

12.) $y'' + 5y' = \cos 10t$, $y(0) = -4$, $y'(0) = 4$.

12A.) $y = -\frac{16}{5} - \frac{99}{125}e^{-5t} - \frac{1}{125}\cos 10t + \frac{1}{250}\sin 10t$.