Learning Catalytics exercise: Here's some space where you can do a calculation:

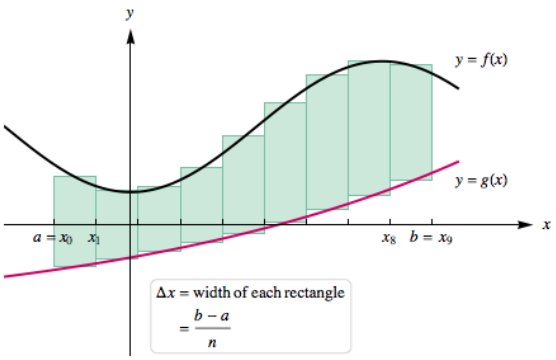
Area Between Two Graphs

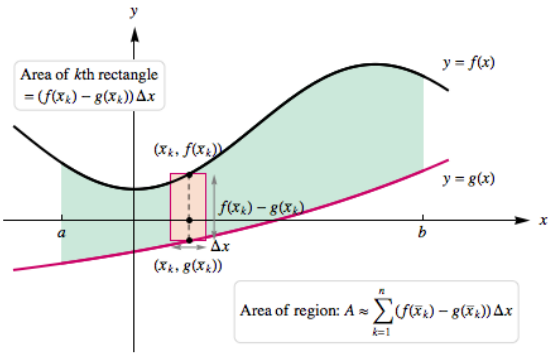
Consider two functions f and g that are continuous on the interval $[a, b]$. Their graphs determine a region in the plane, and we can use the integral to calculate the area of this region.

If $g(x) \leq f(x)$, then it is not difficult to show that the area of this region is

$$\int_a^b f(x) - g(x)\, dx.$$

Figures 6.12 and 6.13 in your textbook suggest why this formula holds.





What happens if $g(x)$ is not always less than $f(x)$? For example, suppose that $g(x) \leq f(x)$ on the interval $[a, b]$ and $f(x) \leq g(x)$ on the interval $[b, c]$ where $a < b < c$.

Then the area between the two graphs is

Example. Find the area of the region between the graphs of the functions $f(x) = x^2 - x$ and $g(x) = x^3 - 4x^2 + 3x$.

Example. Find the area of the region between the graphs of $y = \sin x$ and $y = \cos x$ over the interval $[0, 2\pi]$.

Sometimes it is better to integrate with respect to y rather than with respect to x. In this case, we must express the curves as $x = f(y)$ and $x = g(y)$.

Example. Find the area of the region bounded by the two curves $y = x$ and $x = y^3 - y$.