Calculating Arc Length Using the Definite Integral

Consider the graph \( y = f(x) \) of a differentiable function \( f \). We can use the definite integral to compute its length.
Example. Find the arc length of the graph of $y = \frac{1}{2} (e^x + e^{-x})$ over the interval $0 \leq x \leq \ln 2$. 

![Graph of $y = \frac{1}{2} (e^x + e^{-x})$.
Example. Find the arc length of the graph of $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ over the interval $\frac{1}{2} \leq x \leq 2$.

Most of the time the integrals for arc length cannot be expressed in terms of "elementary" functions.

Example. Calculate the arc length of one hump of the graph of the sine function.
Sometimes it is best if we consider the curve as a function of $y$.

**Example.** Find the arc length of that portion of the curve $6xy - y^4 = 3$ that goes from $(49/48, 1/2)$ to $(14/3, 3)$. 

![Graph of the curve $6xy - y^4 = 3$ with marked points at $(49/48, 1/2)$ and $(14/3, 3)$.]