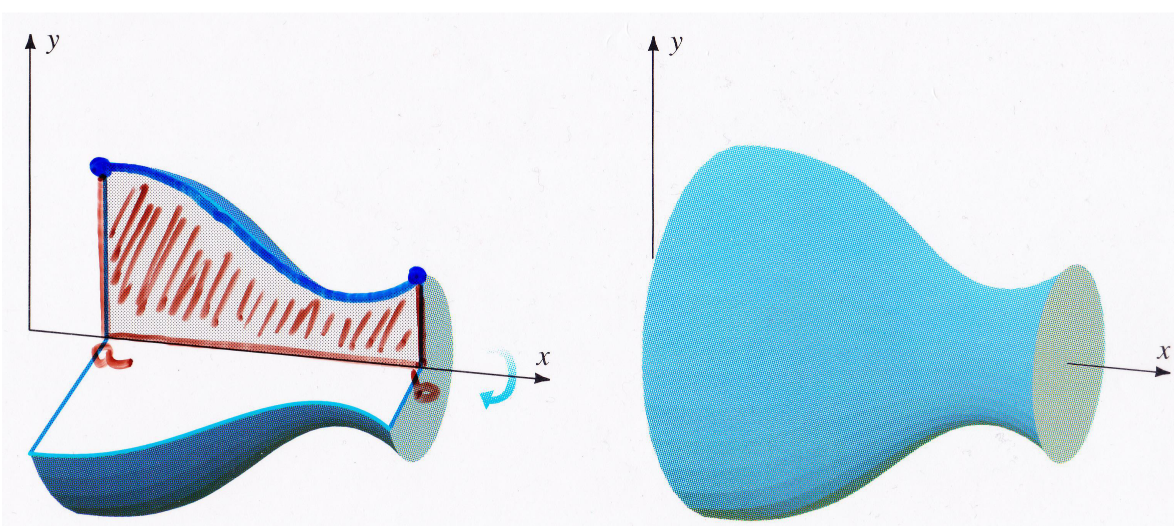
Learning Catalytics exercise: Here's some space where you can do a calculation:

Calculating Volume Using the Definite Integral

Consider the graph $y = f(x)$ of a function. When you revolve this graph about the x-axis, it produces a solid of revolution.

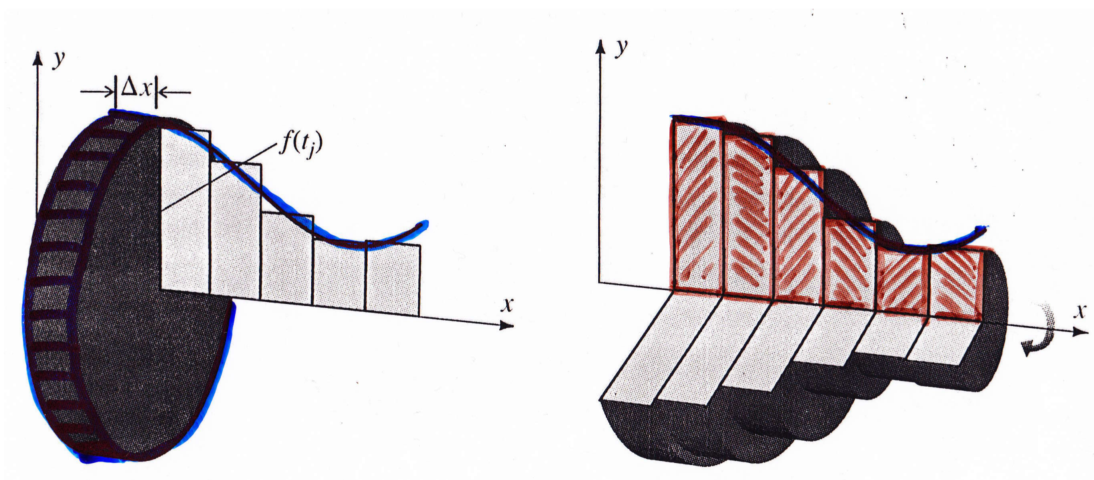


We can calculate the volume of the solid by approximating the volume with Riemann sums. We partition the interval $a \le x \le b$ using equal length subintervals. We get

$$a = x_0 < x_1 < x_2 < \ldots < x_n = b$$

where $x_j - x_{j-1} = \Delta x = (b-a)/n$. In each subinterval $[x_{j-1}, x_j]$, we pick a test number t_j. Then the volume is approximated by the Riemann sum

$$\sum_{j=1}^{n} \pi (f(t_j))^2 \Delta x.$$

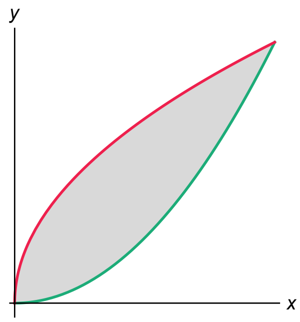


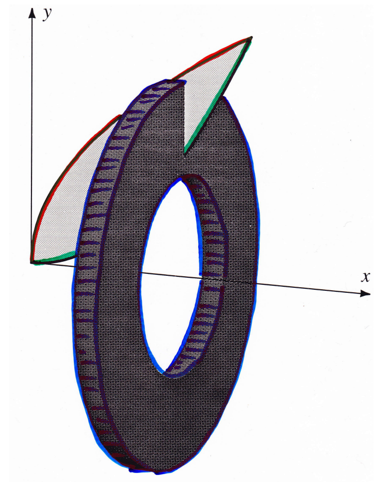
If we let $n \to \infty$, we get the equality

$$\text{volume} = \int_a^b \pi (f(x))^2 \, dx.$$

Example. Let $f(x) = 4x - x^2$. Note that $f(x) = 0$ if $x = 0$ or $x = 4$. Calculate the volume of the solid of revolution if the region bounded by $y = 4x - x^2$ and $y = 0$ is revolved about the x-axis.

We can also consider regions that are determined by two graphs.



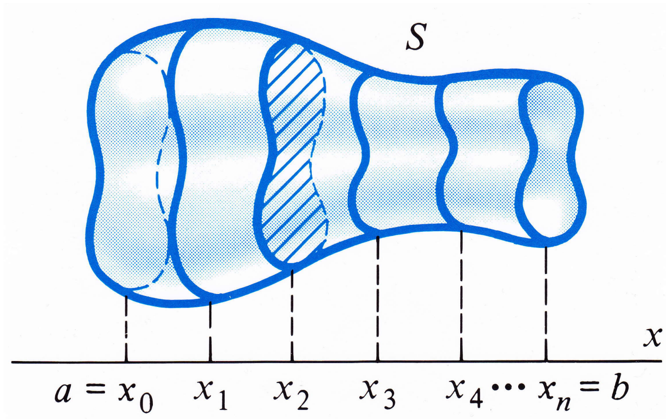


In this case, the integral for volume is

$$\int_a^b \pi(f(x)^2 - g(x)^2)\,dx.$$

Example. Let R be the region bounded by $y = x$ and $y = 2\sqrt{x}$. What is the volume of the solid obtained by revolving R about the x-axis?

These methods are a special case of a more general method for calculating volumes. Suppose that we have a solid, and we know the area of all cross sections perpendicular to a given axis.



Then we can calculate its volume using Riemann sums too. In this situation we see that

$$\text{volume} = \int_a^b A(x)\,dx$$

where $A(x)$ is the function that gives the area of the cross section that is determined by the value of x.

Example. Find the volume of a pyramid that is 50 feet high with a rectangular base that is 50 feet by 100 feet.