Learning Catalytics arc length exercise: Here's some space for you to make a calculation.

Calculating Work Using the Definite Integral

The concept of work in physics is related to the displacement of an object when a force acts upon it. For example, if you pick a calculus book up from the floor, you are applying a force to move it. In the process, "work" is being done on the object.

Work is related to energy. The work-energy theorem states that the change in kinetic energy equals the work done on a rigid body.

For the calculus book example, it makes sense to assume that the force on the book due to gravity is constant. The work done is the product of the force with the distance. For example, if the mass of the book is 1.5 kg and you raise it 1.2 m, then the work done is

$$W = (\text{force})(\text{distance}) = (1.5\text{ g})(1.2),$$

where g is the gravitational constant (9.8 m/s^2).

We need to use the definite integral to compute work when the force is not constant. For example, when we compress a spring, the force involved is not constant, and we cannot simply compute the product of the force and the displacement.

Assume that an object moves along the x-axis from $x = a$ to $x = b$ and a force $f(x)$ acts on the object. We subdivide the interval $a \leq x \leq b$ into n subintervals of equal length (as usual).

Springs: **Hooke's Law** for a simple mass-spring system states that the force required to stretch or compress a spring x units from its equilibrium position is $F(x) = kx$, where k is a constant that measures the stiffness of the spring.

Example. Suppose that a force of 40 N is required to stretch a spring from its natural length of 10 cm to a length of 15 cm. How much work is done stretching the spring from 15 cm to 18 cm? How much work is done compressing the spring from its natural length to a length of 7 cm?

Example. Suppose that a ship's anchor weighs 2 tons (4000 pounds) in water and that the anchor is hanging taut from 100 feet of cable. How much work is required to wind in the anchor if the cable weighs 20 pounds per foot in water?

Example. A water tank has the shape of an inverted circular cone with a height of 5 m and a base radius of 2 m. It is filled with water to a height of 4 m. How much work is required to empty the tank by pumping all of the water to the top of the tank?