Learning Catalytics exercise: Here's some space for you to doodle while you contemplate your answer.

More on integration by parts

Sometimes integration by parts works even when the integral does not seem to be appropriate for the method.

Example. $\int \ln x \, dx$

Integrals of Powers of Trigonometric Functions

We can integrate powers of trig functions using substitution and some of the standard trig identities.

We did some integrals like the following one last semester. See Example 3b in Section 5.5 for a similar example.

Example. $\int \sin x \, \cos^2 x \, dx$

Here is a slightly more complicated integral.

Example. $\int \sin^5 x \, \cos^2 x \, dx$

Remark. If the only powers involved are even, then you must use the half-angle formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \text{and} \quad \cos^2 x = \frac{1 + \cos 2x}{2}.$$

You have already seen how to integrate $\sin^2 x$ and $\cos^2 x$. See Example 6 in Section 5.5.

Example. $\int \sin^2 x \, \cos^2 x \, dx$

We can calculate the integrals

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = \ln |\sec x| + C \quad \text{and} \int \sec x \, dx = \ln |\sec x + \tan x| + C$$

using u-substitution. Other combinations of powers of $\tan x$ and $\sec x$ can be integrated by using the trig identity

$$1 + \tan^2 x = \sec^2 x.$$

Trigonometric Substitutions

Trig substitutions convert integrals involving square roots into integrals that involve powers of trig functions.

Example. We can compute the area of a circle using a trig substitution.