More on trigonometric substitutions

Learning Catalytics exercise: Here's some space for you to make a calculation.

Example. $\int_0^{\pi/2} r^2 \cos^2\theta \, d\theta$

Applying the half-angle formula to $\cos^2\theta$, we have

$$r^2 \int_0^{\pi/2} \left(\frac{1+\cos(2\theta)}{2}\right) d\theta.$$

By the Fundamental Theorem of Calculus, this integral equals

$$r^2 \left[\frac{\theta}{2} + \frac{\sin(2\theta)}{4}\right]_{\theta=0}^{\theta=\pi/2} = r^2 \left[\left(\frac{\pi}{4}+0\right) - \left(0+0\right)\right] = \frac{\pi r^2}{4}.$$

Here is a more complicated indefinite integral.

Example. $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

(There is more space on the next page for converting back to the original variable x.)

The method of partial fractions

This method provides a way to calculate integrals of rational functions. Recall that a rational function is a quotient of two polynomials. For example,

$$\frac{7x+2}{x^2+x-2}$$

is a rational function.

The method of partial fractions is analogous to taking a fraction such as 7/6 and splitting it up into two fractions such as

$$\frac{7}{6} = \frac{1}{2} + \frac{2}{3}.$$

Example. $\int \frac{7x+2}{x^2+x-2}\,dx$

Example. $\int_2^3 \frac{-x+3}{2x^2-5x+3}\,dx = \ln\left(\frac{3^{3/2}}{4}\right)$