Learning Catalytics exercise: Here's some space in case you need to do a quick calculation.

Numerical integration

Many (most?) definite integrals cannot be calculated exactly. For example, we know that *Mathematica* cannot calculate

$$\int_0^1 e^{\cos x}\, dx,$$

but it does know that

$$\int_0^1 e^{\cos x}\, dx \approx 2.34157.$$

How does it make that calculation?

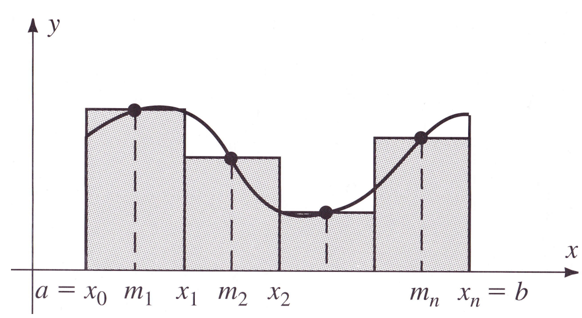
We will discuss three numerical methods for approximating definite integrals—the Midpoint Rule, the Trapezoid Rule, and Simpson's Rule. All three are involve an equal length subdivision of the interval of integration.

Let n be the number of subdivisions. Then $\Delta x = (b-a)/n$, and the subdivision is determined by

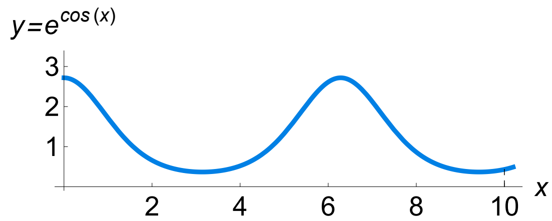
$$a = x_0,\ x_1 = a + \Delta x,\ x_2 = a + 2\Delta x, \ldots,\ x_n = a + n\Delta x = b.$$

The Midpoint Rule evaluates the function at the midpoint m_j of each subinterval $[x_{j-1}, x_j]$.

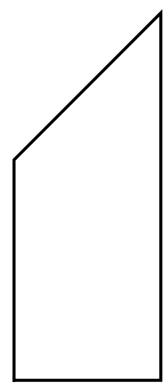
Midpoint Rule:



Example. Using the Midpoint Rule with $n = 5$ subdivisions, approximate $\int_0^{10} e^{\cos x}\, dx$.

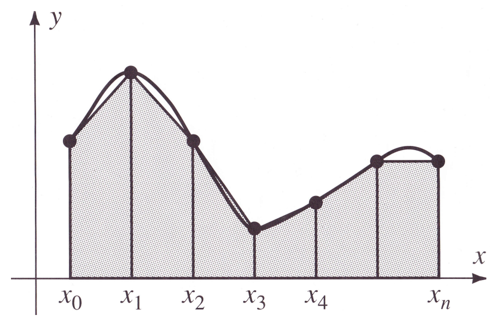


The Trapezoid Rule uses trapezoids rather than rectangles to estimate definite integrals. First, we must recall the area of a trapezoid.

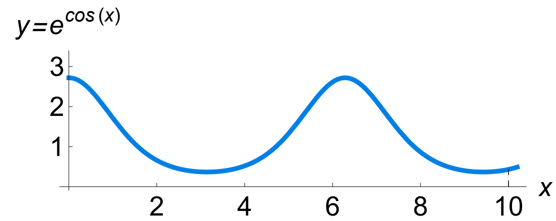


To approximate the $\int_a^b f(x)\, dx$, we again subdivide the interval $[a, b]$ into n subintervals of equal length.

Trapezoid Rule:



Example. Using the Trapezoid Rule with $n = 5$ subdivisions, approximate $\int_0^{10} e^{\cos x}\, dx$.



The following example illustrates how the Trapezoid Rule is used to approximate the integral of a continuously varying quantity given a table of observations of that quantity.

Example. Estimate the average temperature over a four-hour time interval using the observed temperatures listed in the following table:

hour	0	1	2	3	4
temperature	65	69	72	73	71

Hourly temperature readings over a four-hour period.