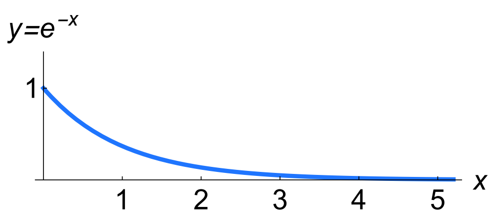
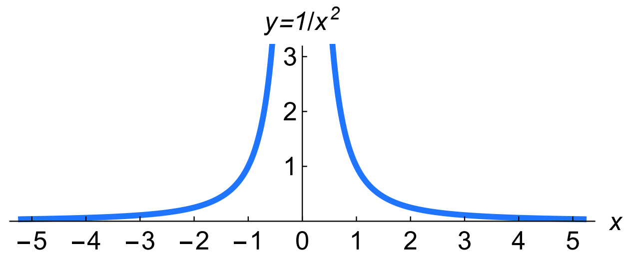
Improper integrals

Improper integrals are integrals with infinite limits of integration such as

$$\int_0^\infty e^{-x}\,dx$$

or integrals over intervals where the function is unbounded such as $\displaystyle\int_{-5}^{5} \frac{1}{x^2}\,dx$.




The physical significance of these types of integrals is illustrated by the following example.

**Example.** Consider a rocket taking off from the Earth. Is there such a thing as escape velocity?

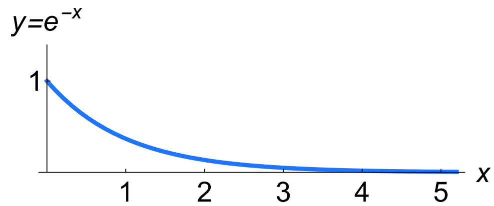
Today we focus on improper integrals on infinite intervals.

**Definition.** Suppose that $f$ is a continuous function on the interval $[a, \infty)$. Then

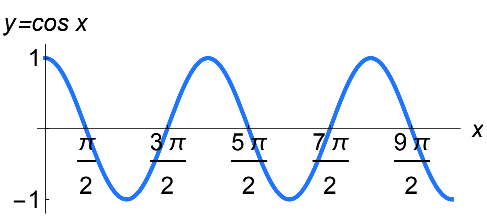
$$\int_a^\infty f(t)\,dt = \lim_{b\to\infty} \int_a^b f(t)\,dt.$$

If this limit exists and is finite, we say that the integral converges. If the limit does not exist or equals either $+\infty$ or $-\infty$, we say that the integral diverges.

**Example.** $\int_0^\infty e^{-x}\,dx$

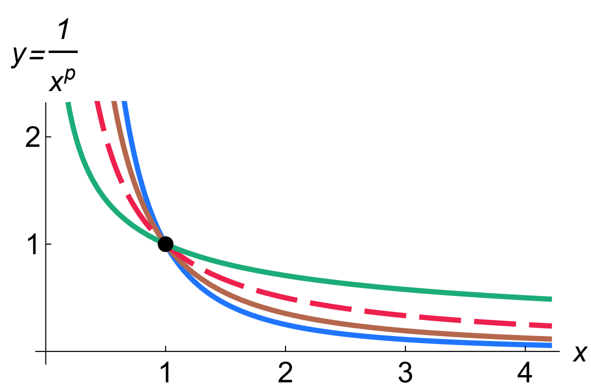



**Example.** $\int_0^\infty \cos x\,dx$



The following example will play a very important role throughout most of the rest of the semester.

**Example.** Consider the "family" of improper integrals $\int_1^{\infty} \frac{1}{x^p}\,dx$ where $p$ is any real constant.

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Here's an interesting comparison between area and volume.

**Example.** Consider the region in the $xy$-plane bounded by the $x$-axis, the line $x = 1$, and the graph of $y = 1/x$.