Learning Catalytics exercise: Here's some space in case you need to do a quick calculation.

The Comparison Test

Theorem. (Comparison Test) Given two series $\sum a_k$ and $\sum b_k$ where the terms a_k and b_k are positive. Then

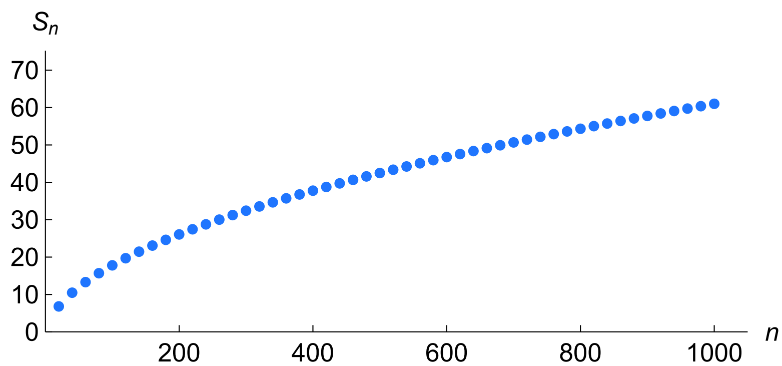
1. If $\sum b_k$ converges and $a_k < b_k$ for all k, then $\sum a_k$ converges.
2. If $\sum a_k$ diverges and $a_k < b_k$ for all k, then $\sum b_k$ diverges.

Example. $\displaystyle\sum_{k=2}^{\infty} \frac{k^3}{k^4-1} = \frac{8}{15} + \frac{27}{80} + \frac{64}{255} + \frac{125}{624} + \dots$

Example. $\displaystyle\sum_{k=2}^{\infty} \frac{\sqrt{k-1}}{k^2+2} = \frac{1}{6} + \frac{\sqrt{2}}{11} + \frac{\sqrt{3}}{18} + \frac{2}{27} + \dots$

Limit Comparison Test

Example. $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k^3}}{1+k^2} = \frac{1}{2} + \frac{\sqrt{8}}{5} + \frac{\sqrt{27}}{10} + \frac{8}{17} + \dots$



Do you think that this series converges?

First try:

Second try:

It would be nice to use a test that does not require that we get the inequalities exactly right.

Theorem. (Limit Comparison Test) Suppose that $\sum a_k$ and $\sum b_k$ are two series with positive terms and

$$\lim_{k\to\infty} \frac{a_k}{b_k} = L.$$

1. If $0 < L < \infty$, then $\sum a_k$ and $\sum b_k$ either both converge or both diverge.
2. If $L = 0$ and $\sum b_k$ converges, then $\sum a_k$ converges.
3. If $L = \infty$ and $\sum b_k$ diverges, then $\sum a_k$ diverges.

Let's consider the preceding example using the Limit Comparison Test.

Example. $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k^3}}{1+k^2} = \frac{1}{2} + \frac{\sqrt{8}}{5} + \frac{\sqrt{27}}{10} + \frac{8}{17} + \dots$

Why is the Limit Comparison Test true?

Example. $\displaystyle\sum_{k=3}^{\infty} \frac{1}{\sqrt{k^3 - k^2 - 2k}} = \frac{1}{\sqrt{12}} + \frac{1}{\sqrt{40}} + \frac{1}{\sqrt{90}} + \dots$

Example. $\displaystyle\sum_{k=2}^{\infty} \frac{\ln k}{k^2} = \frac{\ln 2}{4} + \frac{\ln 3}{9} + \frac{\ln 4}{16} + \dots$