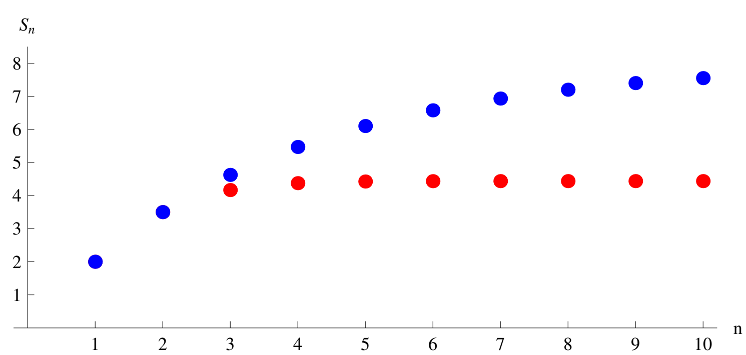
Learning Catalytics exercise: Here's some space in case you need to do a quick calculation.

The Ratio Test

The Ratio Test takes advantage of what we know about geometric series and the Comparison Test.

Example. $\displaystyle\sum_{k=1}^{\infty} \frac{k+1}{k!} = 2 + \frac{3}{2} + \frac{2}{3} + \frac{5}{24} + \dots$



Theorem. (Ratio Test) Let $\sum a_k$ be a series with positive terms and let $r = \lim_{k\to\infty} \frac{a_{k+1}}{a_k}$.

1. If $0 \le r < 1$, then $\sum a_k$ converges.
2. If $r > 1$, then $\sum a_k$ diverges.
3. If $r = 1$, the test is inconclusive.

Example. $\displaystyle\sum_{k=1}^{\infty} \frac{k\,2^{k-1}}{5^k} = \frac{1}{5} + \frac{4}{25} + \frac{12}{125} + \dots$

Example. $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{2^k} = \frac{1}{2} + \frac{1}{2} + \frac{3}{4} + \dots$

Example. What happens when we apply the Ratio Test to a p-series?

The Root Test

Example. $\displaystyle\sum_{k=1}^{\infty} \frac{e^k}{k^k} = e + \frac{e^2}{4} + \frac{e^3}{27} + \frac{e^4}{256} + \dots$

Theorem. (Root Test) Let $\sum a_k$ be a series with positive terms and let

$$\rho = \lim_{k\to\infty} \sqrt[k]{a_k}.$$

1. If $0 \le \rho < 1$, then $\sum a_k$ converges.
2. If $\rho > 1$, then $\sum a_k$ diverges.
3. If $\rho = 1$, the test is inconclusive.

Example. $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{k^2} = \frac{1}{2} + \frac{16}{81} + \frac{19,683}{262,144} + \dots$