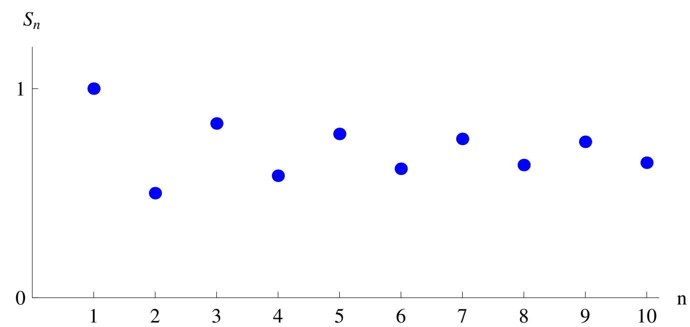
Learning Catalytics exercise: Here's some space in case you need to do a quick calculation.

Alternating series

An alternating series is one in which the terms alternate between positive and negative numbers.

Example. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} \pm \ldots$

Here is a plot of its partial sums:



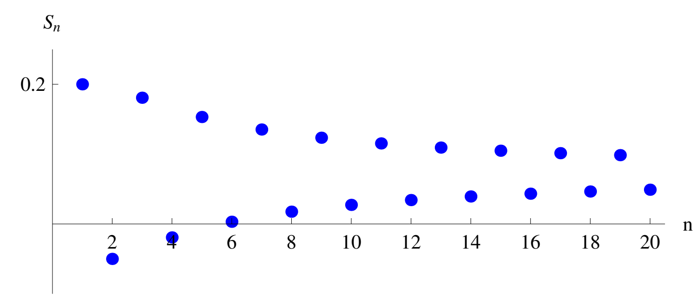
Theorem 1. Consider the alternating series $\sum_{k=1}^{\infty}(-1)^{k+1}a_k$ with $a_k > 0$. It converges if the following two conditions hold:

1. The a_k satisfy the condition that $a_{k+1} \leq a_k$ for all k.
2. $\lim_{k\to\infty} a_k = 0$.

If so, the series converges to a value S between 0 and a_1.

Example. $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k}{k^2+4} = \frac{1}{5} - \frac{1}{4} + \frac{3}{13} - \frac{1}{5} \pm \dots$

Here is a plot of its partial sums:



Estimating the remainder in an alternating series

The nth remainder R_n for a series that converges to S is $R_n = |S - S_n|$.

Theorem 2. For an alternating series that satisfies the hypothesis of Theorem 1, then

$$R_n < a_{n+1}.$$

Example. How large must n be so that the n-th partial sum S_n approximates

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \pm \dots$$

with a remainder that is less than 0.05?