Learning Catalytics exercise: Here's some space in case you need to do a quick calculation.

Estimating the remainder in an alternating series

The nth remainder R_n for a series that converges to S is $R_n = |S - S_n|$.

Theorem 2. For an alternating series that satisfies the hypothesis of Theorem 1, then

$$R_n < a_{n+1}.$$

Example. How large must n be so that the n-th partial sum S_n approximates

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \pm \dots$$

with a remainder that is less than 0.05?

Absolute convergence

Consider the three infinite series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} \pm \dots$$

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36} \pm \dots$$

$$1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + + - \dots$$

We'll also consider a fourth infinite series that we make from the coin tossing that you did before class.

Definition. Let $\sum a_k$ be an infinite series with positive and negative terms. We say that the series converges absolutely if the series

$$\sum |a_k|$$

converges. If $\sum a_k$ converges but does not converge absolutely, then we say that it converges conditionally.

Examples. Consider the three infinite series above.

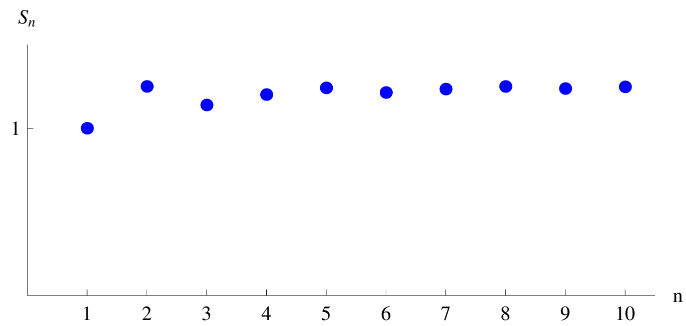
$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} \pm \dots$$

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36} \pm \dots$$

$$1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + + - \dots$$

Here are the partial sums for the third example, that is,

$$1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + + - \dots .$$



Theorem. If the series $\sum a_k$ converges absolutely, then it converges.

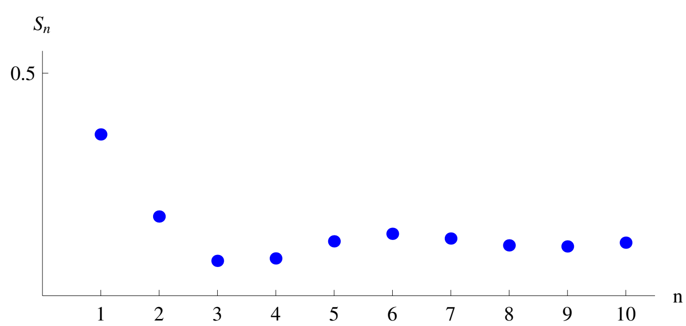
Example. Once again we consider the series

$$1+\frac{1}{4}-\frac{1}{9}+\frac{1}{16}+\frac{1}{25}-\frac{1}{36}++-\dots$$

Example. Consider the infinite series

$$\sum_{k=1}^{\infty}\frac{\cos 1.2k}{k^2}\approx .36-\frac{.74}{4}-\frac{.90}{9}+\frac{.09}{16}+\frac{.96}{25}+\frac{.61}{36}-\frac{.52}{49}-\frac{.99}{64}\dots.$$

Here are the partial sums for this example:



We can modify the Ratio Test so that it tests for absolute convergence.

Theorem. For a series $\sum a_k$ with positive and negative terms, let

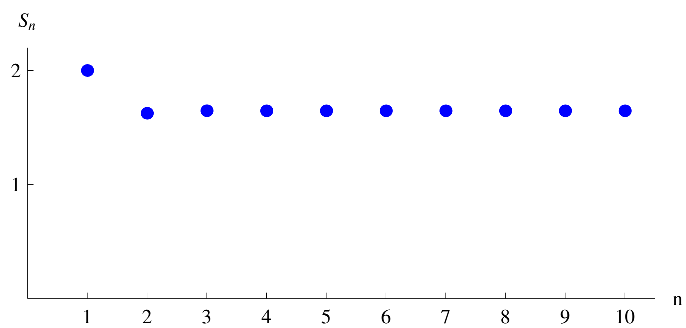
$$r = \lim_{k\to\infty} \frac{|a_{k+1}|}{|a_k|}.$$

If $r < 1$, then the series converges absolutely.

Example. Consider the alternating series

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(k+1)^2}{(2k)!} = 2 - \frac{3}{8} + \frac{1}{45} - \frac{5}{8064} \pm \dots$$

Here are the partial sums for this example:



Crazy Fact: If a series converges absolutely, then its terms may be summed in any order without changing the value of the series. However, if the series converges conditionally, then the value of the series depends on the order of the terms.