More on the binomial series

We need a generalization of the "p choose k" notation.

Definition. Given a real number p and a positive integer k,

$$\binom{p}{k} = \frac{(p)(p-1)(p-2)\cdots(p-k+1)}{k!} \quad \text{and} \quad \binom{p}{0} = 1.$$

Learning Catalytics exercise: Here's some space in case you need to do a quick calculation or want to take some notes when we finish the exercise.

At the end of last class, we discussed binomial expansion. We recalled Pascal's triangle and the binomial formula

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n-1}ab^{n-1} + b^n$$

where

$$\binom{n}{k} = \frac{(n)(n-1)(n-2)\cdots(n-k+1)}{k!}.$$

If we factor out a and let $x = b/a$, we have

$$(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots + \binom{n}{k}x^k + \ldots + x^n.$$

Newton (among others) realized that we can use Taylor series to compute the values of $f(x) = (1+x)^p$ for any real value of p. For example, we can use Taylor series to compute roots such as square roots ($p = 1/2$) and cube roots ($p = 1/3$).

We obtain the binomial series by calculating the Maclaurin series for the function

$$f(x) = (1+x)^p$$

where p is any real number.

Theorem. Given a real number p, the power series

$$\sum_{k=0}^{\infty} \binom{p}{k} x^k = \sum_{k=0}^{\infty} \frac{(p)(p-1)(p-2)\cdots(p-k+1)}{k!}\, x^k$$

converges to the function $f(x) = (1+x)^p$ for $|x| < 1$.

Example. Compute the binomial series for $f(x) = \dfrac{1}{\sqrt{1+x}}$.

Example. Use the first four terms of a binomial series to approximate $\sqrt[5]{1.1}$