Learning Catalytics exercise: Here's some space in case you need to do a quick calculation or want to take some notes when we finish the exercise.

Differential equations

Differential equations appear in numerous applications of calculus. For example, they describe the decay of a radioactive material, the motion of the planets, and the growth of a population.

We have already encountered some differential equations. We solve a differential equation every time we calculate an antiderivative. We also solved the differential equation

$$\frac{dy}{dt} = ky$$

when we discussed exponential models. Now we consider a more varied collection of first-order differential equations.

Definition. A first-order differential equation is an equation of the form

$$\frac{dy}{dt} = f(t, y)$$

where t is the independent variable and y is the dependent variable.

Solving initial-value problems:

Be careful about notation:

Separable differential equations

A differential equation $dy/dt = f(t, y)$ is **separable** if it can be written in the form

$$\frac{dy}{dt} =$$

Two Examples:

1. $\dfrac{dy}{dt} = -2ty^2$

2. $\dfrac{dy}{dt} = y^3 + t^2$

We can solve separable differential equations using the method of substitution. Consider the first example

Example. $\dfrac{dy}{dt} = -2ty^2$

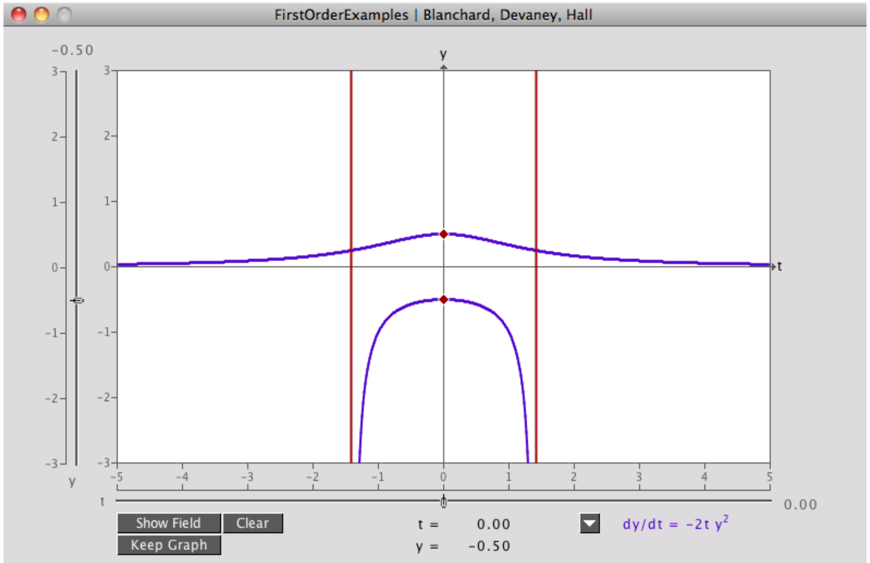
Let's solve the initial-value problem

$$\frac{dy}{dt} = -2ty^2, \quad y(0) = \frac{1}{2}.$$

Compare that solution to the solution of the initial-value problem

$$\frac{dy}{dt} = -2ty^2, \quad y(0) = -\frac{1}{2}.$$

We turn to the computer to get a sense of the graphs of these solutions:



An important type of separable equation is the equation

$$\frac{dy}{dt} = ky + b$$

where k and b are constants. The constant b represents a growth or decay rate that is due to external factors. This particular equation is an example of a first-order linear differential equation. It is also one that can be solved using separation of variables.

Example. A cup of hot chocolate that is initially 120° sits in a 70° degree room. Newton's Law of Cooling states that the rate at which it cools is proportional to the difference between its current temperature and the ambient temperature (in this case, 70°). Suppose that the hot chocolate is cooling at the rate of 10° per minute at time $t = 0$. How long does it take for it to cool to 80°?