Learning Catalytics exercise: Here's some space in case you need to do a quick calculation or want to take some notes when we finish the exercise.

A Linear Differential Equation

An important type of separable equation is the equation

$$\frac{dy}{dt} = ky + b$$

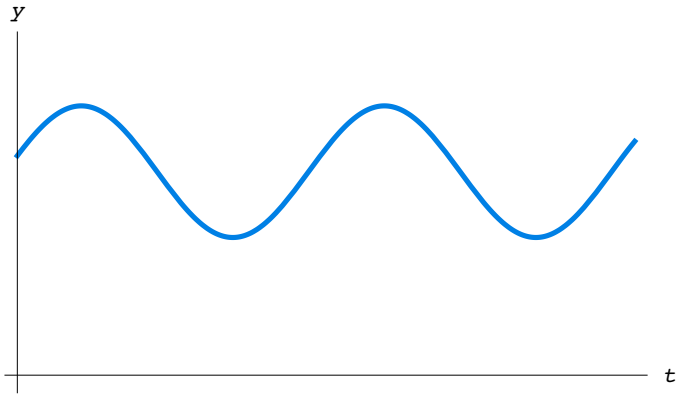
where k and b are constants. The constant b represents a growth or decay rate that is due to external factors. This particular equation is an example of a first-order linear differential equation. It is also one that can be solved using separation of variables.

Example. A cup of hot chocolate that is initially 120° sits in a 70° degree room. Newton's Law of Cooling states that the rate at which it cools is proportional to the difference between its current temperature and the ambient temperature (in this case, 70°). Suppose that the hot chocolate is cooling at the rate of 10° per minute at time $t = 0$. How long does it take for it to cool to 80°?

Slope fields

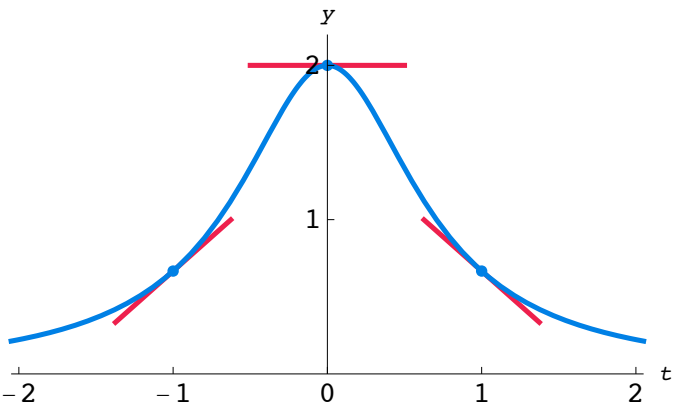
A **slope field** in the ty-plane is a picture of a first-order differential equation

$$\frac{dy}{dt} = f(t, y).$$



The graph of a solution must be everywhere tangent to the slope field.

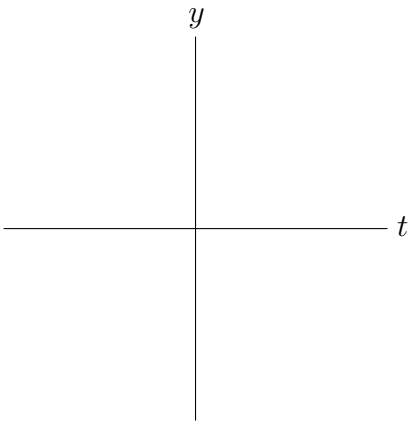
Example. Once again consider the differential equation $\dfrac{dy}{dt} = -2ty^2$.



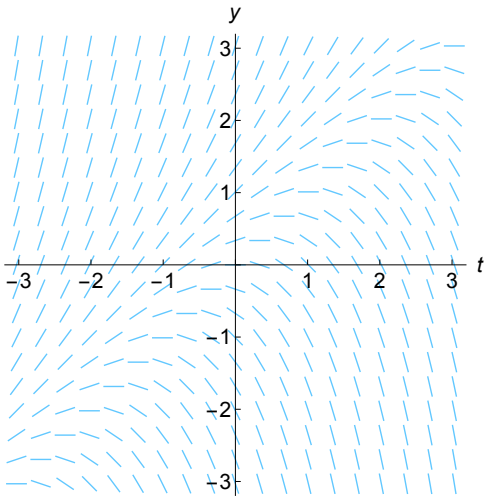
Example. Consider the differential equation

$$\frac{dy}{dt} = y - t.$$

(t, y)	$f(t, y) = y - t$
$(0, 0)$	0
$(1, 0)$	
$(0, 1)$	
$(-1, 0)$	



Using the computer to plot the slope field, we get



The general solution of this differential equation is $y(t) = 1 + t + ce^t$. We can always check:

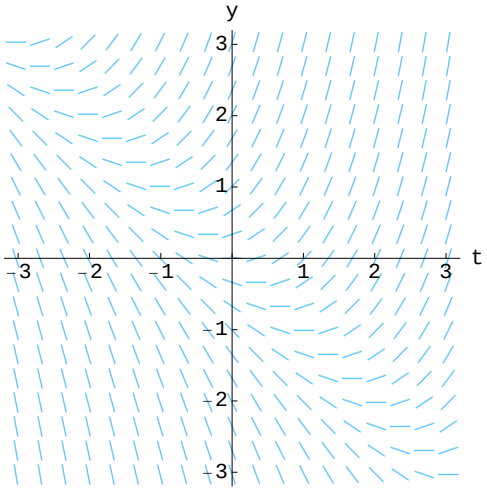
Typical exam problem:

Consider the following 8 first-order equations:

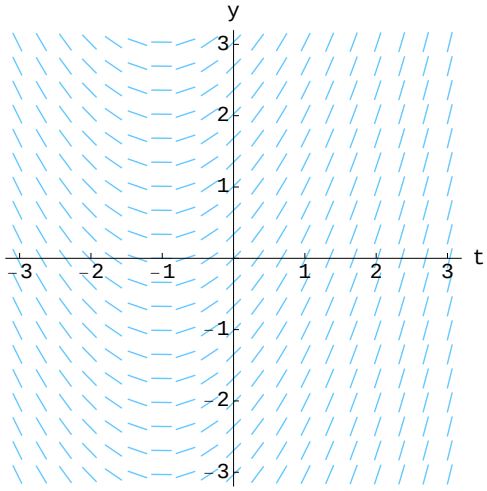
1. $\frac{dy}{dt} = t - 1$
2. $\frac{dy}{dt} = t + 1$
3. $\frac{dy}{dt} = y + 1$
4. $\frac{dy}{dt} = 1 - y$
5. $\frac{dy}{dt} = y^2 + y$
6. $\frac{dy}{dt} = y(y^2 - 1)$
7. $\frac{dy}{dt} = y - t$
8. $\frac{dy}{dt} = y + t$

Four of the associated slope fields are shown below. Pair the slope fields with their associated equations. Provide a brief justification for your choice. (Hint: Look carefully at the t-axis in Fields C and D.)

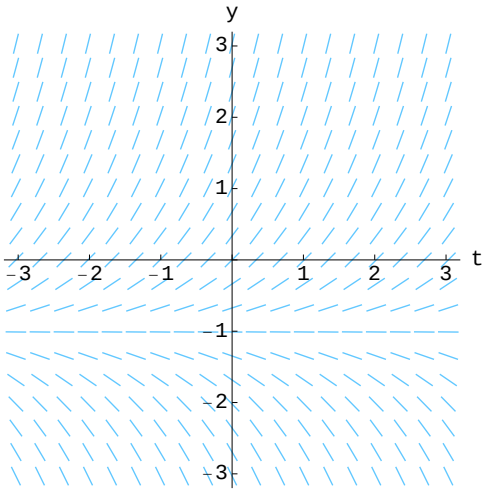
Slope Field A



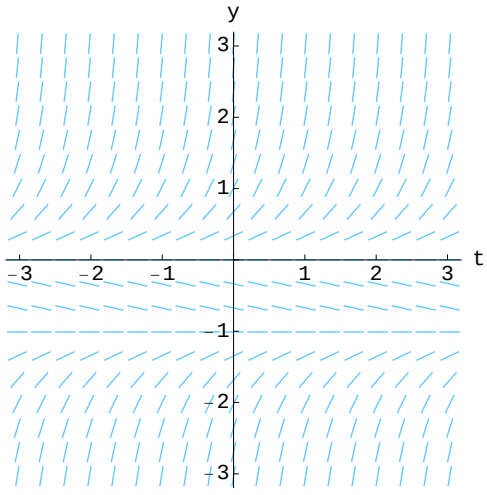
Slope Field B



Slope Field C



Slope Field D



Here's the slope field and the graph of the solution that corresponds to the hot chocolate example that we discussed at the start of class.