Working with Taylor series

Sometimes we can use power series to calculate limits, to solve differential equations, and to estimate. We begin with a limit.

**Example.** Calculate \( \lim_{x \to 0} \frac{3 \tan^{-1} x - 3x + x^3}{x^5} \).
We can solve certain differential equations using power series, but before we do, let’s calculate the Maclaurin series for \( f(t) = e^{2t} \).

**Example.** Compute the Maclaurin series for \( e^{2t} \).

**Example.** Use power series to solve the initial-value problem

\[
\frac{dy}{dt} = 2y + 6, \quad y(0) = 5.
\]
We can use power series to approximate definite integrals.

**Example.** Estimate the integral $\int_{0}^{1} e^{-x^2} \, dx$ with an error no larger than 0.01.
Example. Find the function represented by the series

$$\sum_{k=1}^{\infty} \frac{kx^{k-1}}{3^k} = \frac{1}{3} + \frac{2x}{9} + \frac{x^2}{9} + \frac{4x^3}{81} + \ldots$$