More work with Taylor series

We start with another power series solution to a differential equation.

**Example.** Simple mass-spring system

Hooke’s Law: The restoring force of the spring is proportional to the displacement from its rest position.

Using Newton’s law $F = ma$, we get

$$\frac{d^2 y}{dt^2} = -2y,$$

and we can use power series to produce some solutions.
We can use power series to approximate definite integrals.

**Example.** Estimate the integral \( \int_{0}^{1} e^{-x^2} \, dx \) with an error no larger than 0.01.
Example. Find the function represented by the series

\[ \sum_{k=1}^{\infty} \frac{kx^{k-1}}{3^k} = \frac{1}{3} + \frac{2x}{9} + \frac{x^2}{9} + \frac{4x^3}{81} + \ldots \]
We can multiply and divide power series as if they are infinitely long polynomials.

**Example.** Calculate the first three nonzero terms in the Maclaurin series for $e^x \sin x$. 