1. (21 points) Calculate:

(a) The length of the projection of the vector \( \mathbf{A} = \mathbf{i} + \mathbf{j} - 2\mathbf{k} \) onto the vector \( \mathbf{B} = -2\mathbf{i} - \mathbf{j} + \mathbf{k} \).

\[
\text{length} = \left| \text{comp}_\mathbf{B} \mathbf{A} \right| = \left| \frac{\mathbf{A} \cdot \mathbf{B}}{\left| \mathbf{B} \right|} \right| = \frac{|-5|}{\sqrt{4 + 1 + 1}} = \frac{5}{\sqrt{6}}
\]

(b) A vector equation for the line that contains the point \((2, -1, 3)\) and is perpendicular to the plane \(x + 3y - 2z = 4\).

\[
\overrightarrow{\text{dir vector for line}} \quad \overrightarrow{\mathbf{D}} = t\mathbf{i} + 3t\mathbf{j} - 2t\mathbf{k}
\]

\[
\overrightarrow{l}(t) = (2 + t)\mathbf{i} + (-1 + 3t)\mathbf{j} + (3 - 2t)\mathbf{k}
\]

(c) The area of the triangle in space with vertices \(P = (0, 1, 0), Q = (4, 1, -2),\) and \(R = (5, 3, 1)\).

\[
\overrightarrow{\mathbf{PQ}} = 4\mathbf{i} - 2\mathbf{k} \\
\overrightarrow{\mathbf{PR}} = 5\mathbf{i} + 2\mathbf{j} + \mathbf{k}
\]

\[
\text{area triangle} = \frac{1}{2} \left| \overrightarrow{\mathbf{PQ}} \times \overrightarrow{\mathbf{PR}} \right|
\]

\[
\overrightarrow{\mathbf{PQ}} \times \overrightarrow{\mathbf{PR}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & -2 \\ 5 & 2 & 1 \end{vmatrix} = 4\mathbf{i} - 14\mathbf{j} + 8\mathbf{k}
\]

\[
\text{area} = \frac{1}{2} \sqrt{16 + 196 + 64} = \frac{\sqrt{276}}{2}
\]
2. (18 points) Let 

\[ \mathbf{r}(t) = (3t^2) \mathbf{i} + (te^t) \mathbf{j} + (\ln t) \mathbf{k}, \]

and suppose that \( \mathbf{r}(t) \) is the position function of a particle moving through space. Calculate the velocity, acceleration, and speed of the particle when it is at the point \((3, e, 0)\).

\[ \mathbf{r}(t) = 3t \mathbf{i} + e^t \mathbf{j} \quad \iff \quad \begin{align*}
3t^2 &= 3 & \text{for } t = 1 \\
\ln t &= e & \text{for } t = 1 \\
lnt &= 0 & \Rightarrow t = 1
\end{align*} \]

\[ \mathbf{r}'(t) = (6t) \mathbf{i} + (te^t + e^t) \mathbf{j} + \left(\frac{1}{t}\right) \mathbf{k} \]

\[ = (6t) \mathbf{i} + (t+1)e^t \mathbf{j} + \left(\frac{t+1}{t}\right) \mathbf{k} \]

\[ \mathbf{r}''(t) = 6 \mathbf{i} + \left(\frac{et + (t+1)e^t}{(t+1)e^t}\right) \mathbf{j} + \left(-\frac{1}{t^2}\right) \mathbf{k} \]

Velocity at \((3, e, 0)\):

\[ \mathbf{r}'(1) = 6 \mathbf{i} + (2e) \mathbf{j} + \mathbf{k} \]

acceleration at \((3, e, 0)\):

\[ \mathbf{r}''(1) = 6 \mathbf{i} + 3e \mathbf{j} - \mathbf{k} \]

speed at \((3, e, 0)\):

\[ |\mathbf{r}'(1)| = \sqrt{36 + 4e^2 + 1} = \sqrt{37 + 4e^2} \]
3. (20 points) Find the area of the region in the first quadrant that is inside the circle
\( r = 3 \cos \theta \) and below the line \( y = x \). (The half-angle formula for cosine is
\( \cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta) \).)

\[
y = x \quad \text{first quadrant}
\Leftrightarrow \quad \theta = \frac{\pi}{4}
\]

\[
\text{area} = \frac{1}{2} \int_0^{\pi/4} r^2 \, d\theta = \frac{1}{2} \int_0^{\pi/4} (3 \cos \theta)^2 \, d\theta
\]
\[
= \frac{9}{2} \int_0^{\pi/4} \cos^2 \theta \, d\theta
\]
\[
= \frac{9}{4} \int_0^{\pi/4} (1 + \cos 2\theta) \, d\theta
\]

Let \( u = 2\theta \Rightarrow du = 2 \, d\theta \)
\( \theta = 0 \Rightarrow u = 0 \)
\( \theta = \pi/4 \Rightarrow u = \pi/2 \)

\[
\text{area} = \frac{9}{4} \int_0^{\pi/2} (1 + \cos u) \left(\frac{1}{2}\right) \, du
\]
\[
= \frac{9}{8} \left[ u + \sin u \right]_0^{\pi/2}
\]
\[
= \frac{9}{8} \left[ \frac{\pi}{2} + 1 \right]
\]
4. (18 points) The two lines

\[ \mathbf{L}_1(t) = (1 + t) \mathbf{i} + (1 - 2t) \mathbf{j} + (3t) \mathbf{k} \]

and

\[ \mathbf{L}_2(t) = (2 + t) \mathbf{i} + (11 + 2t) \mathbf{j} + (-3 + t) \mathbf{k} \]

intersect.

(a) Where do they intersect? Use \( s \) as the parameter for \( \mathbf{L}_2 \).

\[
\begin{align*}
1 + t &= 2 + s 
\Rightarrow t &= s + 1 \\
1 - 2t &= 11 + 2s
\Rightarrow 1 - 2(s + 1) = 11 + 2s
\Rightarrow 4s = -12
\Rightarrow s &= -3 \\
3t &= -3 + t 
\text{(check: } -6 \neq -6) 
\Rightarrow t &= -2
\end{align*}
\]

\[ \mathbf{L}_1(-2) = -2 \mathbf{i} + 5 \frac{3}{2} \mathbf{j} - 6 \mathbf{k} \]

\[
(-1, 5, -6)
\]

(b) Calculate the angle of intersection.

\[ \mathbf{D}_1 = \text{dir vector} \mathbf{L}_1 = t - 2 \mathbf{j} + 3 \mathbf{k} \]

\[ \mathbf{D}_2 = \text{dir vector} \mathbf{L}_2 = t + 2 \mathbf{j} + \mathbf{k} \]

\[ \cos \theta = \frac{\mathbf{D}_1 \cdot \mathbf{D}_2}{|\mathbf{D}_1||\mathbf{D}_2|} = \frac{0}{|\mathbf{D}_1||\mathbf{D}_2|} = 0 \]

\[ \theta = \pi/2 \] radians

(c) Find an equation for the plane that contains these two lines.

\[ \mathbf{N} = \text{normal vec} = \mathbf{D}_1 \times \mathbf{D}_2 \]

\[ \mathbf{N} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & -2 & 3 \\
1 & 2 & 1
\end{vmatrix} = -8 \mathbf{t} + 2 \mathbf{j} + 4 \mathbf{k} \]

\[ \mathbf{N} \cdot \mathbf{PQ} = 0 \Rightarrow -8(x+1) + 2(y-5) + 4(z+6) = 0 \]

\[ -8x + 2y + 4z = -6 \text{ or } -4x + y + 2z = -3. \]
5. (20 points) Here are two surfaces in space:

Here are 6 equations of surfaces:

\[ z^2 = x^2 + y^2 + 1 \quad \rightarrow \quad 1. \quad z^2 - x^2 - y^2 = 1 \]

Plane \[ \rightarrow \quad 3. \quad 2x + y - z = 3 \]

\[ z = x^2 + y^2 \quad \rightarrow \quad 5. \quad x^2 + y^2 - z = 0 \]

For each surface, pick the equation that describes it. Provide a brief justification for your choice. You will not receive any credit unless you provide a valid justification.

(a) The equation for surface A is \( 4 \). My reason for choosing this answer is:

The equation for \# 4 is \( y = x^2 + z^2 \). Therefore, \( y \) is never negative, and slices by planes \( y = k > 0 \) are circles. The \( x = 0 \) slice is the parabola \( y = z^2 \), and the \( z = 0 \) is the parabola \( y = x^2 \).

(b) The equation for surface B is \( 4 \). My reason for choosing this answer is:

From \( z^2 = x^2 + y^2 + 1 \), we see that either \( z \geq 1 \) or \( z \leq -1 \). The horizontal slices \( z = k \) are circles of radius \( \sqrt{k^2 - 1} \).

The \( x = 0 \) slice is the hyperbola \( z^2 - y^2 = 1 \), and the \( y = 0 \) slice is the hyperbola \( z^2 - x^2 = 1 \).