More on parametric surfaces and 3D coordinate systems

Last class we discussed what a parametric surface is, and we parametrized the elliptic cylinder

$$x^2 + \frac{y^2}{4} = 1$$

by the vector-valued function $\mathbf{r}(\theta, z) = (\cos\theta)\,\mathbf{i} + (2\sin\theta)\,\mathbf{j} + z\,\mathbf{k}$. There is a link to a graphing applet for parametric surfaces on the course web page along with a *Mathematica* notebook that illustrates how parametric surfaces are plotted within *Mathematica*. Today we consider parameterizations that are based on nonrectangular 3D coordinate systems.

In addition to rectangular coordinates in space, there are two other coordinate systems that we frequently use.

Cylindrical Coordinates

Cylindrical coordinates consist of polar coordinates in the xy-plane along with the usual rectangular coordinate z. Unlike polar coordinates, we often restrict our attention to the situation where $r \geq 0$.

Example. Use cylindrical coordinates to parametrize the hyperboloid of one sheet

$$x^2 + y^2 - z^2 = 1.$$

Spherical coordinates

Another 3D coordinate system that is often convenient to use is the spherical coordinate system.

Example. Parametrize the unit sphere using spherical coordinates.

Example. Parametrize the elliptical cone

$$x^2 + \frac{y^2}{4} - z^2 = 0$$

using spherical coordinates.

MA 225 Exam Logistics

1. Bring pen/pencil and id. You may use your calculator if you wish. If you have a calculator that does symbolic derivatives/integrals, you should make sure that your answer shows that you could do the problem without your calculator.

2. Closed book exam. No extra papers. No ipods, cell phones, etc.

3. Exam will start promptly at 10:00 and end at 10:50.

4. We will collect exams by moving up the aisles. You must pass in your exam when we arrive at your aisle. Please remain seated and quiet until we collect the exams from your aisle.

5. Five minute rule will be in effect: No one will be allowed to leave the exam between 10:45 and 10:50. Use those 5 minutes to check your work.

6. Seating will be assigned before the exam starts.

7. If you have a question, raise your hand. Stay seated.

8. Go to the bathroom before the exam starts.