Using cylindrical and spherical coordinates

Cylindrical and spherical coordinates can be used to simplify many triple integrals that possess radial or spherical symmetries.

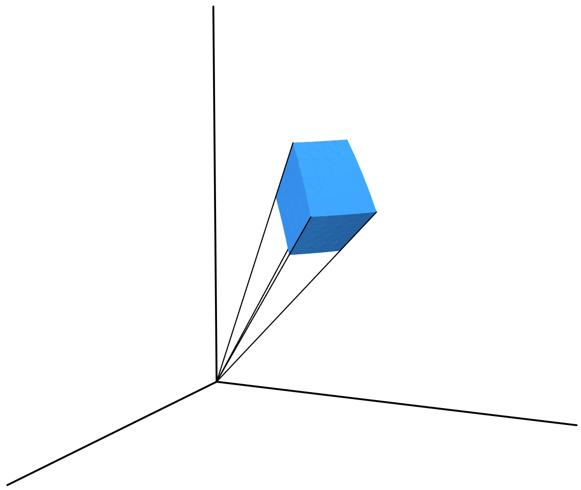
Cylindrical coordinates (r, θ, z) are used when the region and function are best expressed in terms of polar coordinates in x and y.

Marilyn Vos Savant problem: Pick a sphere ... any sphere. Bore a perfect hole through the center in such a way that the remaining solid is 6 inches high. What is the volume that remains?

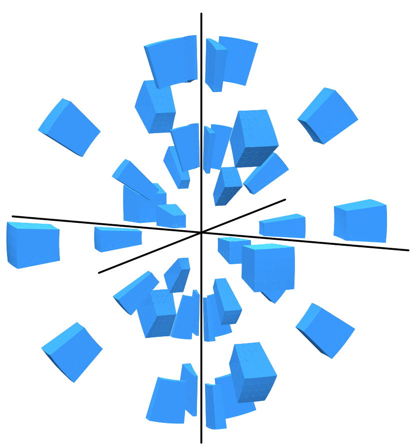
Converting integrals to spherical coordinates

Whenever we use a new coordinate system to do a triple integral, we need a volume adjustment factor. In other words, we need to determine how the volume of a "cube" in the new coordinates converts to a volume in rectangular coordinates.

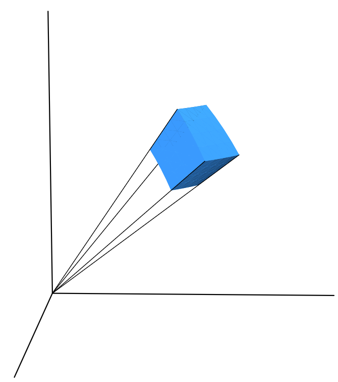
Consider a "spherical cube" with sides $\Delta\rho$, $\Delta\phi$, and $\Delta\theta$.



If we look at various cubes all with the same $\Delta\rho$, $\Delta\phi$, and $\Delta\theta$, we see that their volumes vary greatly.



Let's see if we can approximate this variation.



In summary, our volume conversion factor for spherical coordinates is

$$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta.$$

Example. Let R be the region inside the first octant as well as inside the sphere of radius 2 centered at the origin. Let's evaluate

$$\iiint_R e^{(x^2+y^2+z^2)^{3/2}} \, dV.$$