

Surfaces

The precise mathematical definition of a surface in space is technical and complicated. For our purposes, the following simpler statement will suffice.

Definition. A *surface* in space is a “two-dimensional” collection of points in space.

The meaning of the term “two-dimensional” is best illustrated by a few examples.

Examples.

1. Any plane $ax + by + cz = d$ is a surface.

2. The boundary of any solid region is a surface, e.g., the collection of points satisfying the inequality

$$x^2 + y^2 + z^2 \leq 1$$

has the spherical surface

$$x^2 + y^2 + z^2 = 1$$

as its boundary.

3. Take the equation of any curve in the xy -plane, e.g., the ellipse

$$2x^2 + y^2 = 1,$$

and plot all points (x, y, z) such that

$$2x^2 + y^2 = 1.$$

The result is a surface that is perpendicular to the xy -plane. This type of surface is called a (generalized) *cylinder*.

4. Surfaces of Revolution. Start with the graph of a function of one variable $z = f(y)$ in the yz -plane. Then we can revolve that curve around the y -axis in space. We get a surface of revolution.

5. Graph of a function of two variables. Given a “nice” function $z = f(x, y)$, then the set of all points (x, y, z) such that $z = f(x, y)$ is a surface.

Example. Consider the function $f(x, y) = x^2 + y^2$.

6. The Mobius Band. Take a long and relatively thin strip of paper and attach the two short ends by a half twist.

7. Surfaces from torus knots

8. Carin Siegerman surface

9. Quadric Surfaces.

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz + G = 0.$$

One way to analyze a surface is by studying its traces.

Example. Consider the surface

$$x^2 + \frac{y^2}{4} - \frac{z^2}{9} = 1.$$

Suppose we intersect the surface with the plane $z = k$.

Given any equation for a quadric surface, I want you to be able to sketch it using the methods I described here. The slices are conic sections. You should know the names of the different types and what the differences are. There are six types.

- (a) Ellipsoid
- (b) Elliptic Paraboloid
- (c) Elliptic Cone
- (d) Hyperboloid of One Sheet
- (e) Hyperboloid of Two Sheets
- (f) Hyperbolic Paraboloid

See p. 682 of your textbook.