

Triple integral done as an x -simple region

In class we spent quite a bit of time discussing the following example.

Example. Evaluate

$$\iiint_Q z \, dV$$

where Q is the region bounded by the cylinder $x^2 + z^2 = 9$, the plane $y + z = 3$, and the plane $y = 0$.

When we treated the region as y -simple, we obtained the integral

$$\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{3-z} z \, dy \, dz \, dx = -\frac{81}{4}\pi.$$

Since the region is also x -simple, this integral can also be expressed as

$$\int_0^6 \int_{-3}^{3-y} \int_{-\sqrt{9-z^2}}^{\sqrt{9-z^2}} z \, dx \, dz \, dy.$$