More on triple integrals

Last class we spent quite a bit of time discussing the following example.

Example. Evaluate
$$\iiint\limits_Q z\,dV$$
where Q is the region bounded by the cylinder $x^2+z^2=9$, the plane $y+z=3$, and the plane $y=0$.

When we treated the region as y-simple, we obtained the integral
$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_0^{3-z} z\,dy\,dz\,dx = -\frac{81}{4}\pi.$$

To calculate this iterated integral, it is very helpful to convert the outermost double integral to polar coordinates in the xz-plane.

We can also view this region as x-simple and evaluate the integral that way. Try setting up the integral where $dV = dx\,dz\,dy$. In other words,
$$\iiint\limits_Q z\,dV = \int \int \quad \int \qquad z\,dx\,dz\,dy.$$

The answer is posted on the course web site.

Note that, in triple integrals,

1. the outermost limits of integration are constants,
2. the intermediate limits of integration only depend on the outermost variable, and
3. the intermost limits of integration only depend on the two outermost variables.

Using cylindrical and spherical coordinates

Cylindrical and spherical coordinates can be used to simplify many triple integrals that possess radial or spherical symmetries.

Cylindrical coordinates (r,θ,z) are used when the region and function are best expressed in terms of polar coordinates in x and y.

Let's reorient the solid Q and then calculate its volume using cylindrical coordinates.

Example. Let Q be the solid region bounded by the cylinder $x^2+y^2=9$, the plane $y+z=3$, and the plane $z=0$. We can calculate its volume by calculating

$$\iiint\limits_Q 1\,dV,$$

and then we'll know the z-coordinate of its centroid.

Marilyn Vos Savant problem: Pick a sphere ... any sphere. Bore a perfect hole through the center in such a way that the remaining solid is 6 inches high. What is the volume that remains?

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