Surfaces

The precise mathematical definition of a surface in space is technical and complicated. For our purposes, the following simpler statement will suffice.

**Definition.** A *surface* in space is a “two-dimensional” collection of points in space.

The meaning of the term “two-dimensional” is best illustrated by a few examples.

**Examples.**

1. Any plane \( ax + by + cz = d \) is a surface.

2. The boundary of any solid region is a surface, e.g., the collection of points satisfying the inequality

   \[ x^2 + y^2 + z^2 \leq 1 \]

   has the spherical surface

   \[ x^2 + y^2 + z^2 = 1 \]

   as its boundary.
3. Take the equation of any curve in the $xy$-plane, e.g., the ellipse
\[ 2x^2 + y^2 = 1, \]
and plot all points $(x, y, z)$ such that
\[ 2x^2 + y^2 = 1. \]
The result is a surface that is perpendicular to the $xy$-plane. This type of surface is
called a (generalized) cylinder.

4. Surfaces of Revolution. Start with the graph of a function of one variable $z = f(y)$ in
the $yz$-plane. Then we can revolve that curve around the $y$-axis in space. We get a
surface of revolution.
5. Graph of a function of two variables. Given a “nice” function $z = f(x, y)$, then the set of all points $(x, y, z)$ such that $z = f(x, y)$ is a surface.

**Example.** Consider the function $f(x, y) = x^2 + y^2$. 
6. The Mobius Band. Take a long and relatively thin strip of paper and attach the two short ends by a half twist.

7. Surfaces from torus knots: This figure was produced using computer code posted on the web site of Professor Mark McClure at the University of North Carolina in Asheville.

8. Carin Siegerman surface: In class I will show a figure that can be interpreted as a surface. It was produced by Carin Siegerman, a former MA225 student of mine, as part of her work on her PhD dissertation in biomedical engineering.

9. Quadric Surfaces: surfaces of the form

\[ Ax^2 + By^2 + Cz^2 = D \]

where \( A, B, \) and \( C \) are nonzero.