Surfaces

The precise mathematical definition of a surface in space is technical and complicated. For our purposes, the following simpler statement will suffice.

Definition. A *surface* in space is a "two-dimensional" collection of points in space.

The meaning of the term "two-dimensional" is best illustrated by a few examples.

Examples.

1. Any plane $ax + by + cz = d$ is a surface.

2. The boundary of any solid region is a surface, e.g., the collection of points satisfying the inequality
$$x^2 + y^2 + z^2 \leq 1$$
has the spherical surface
$$x^2 + y^2 + z^2 = 1$$
as its boundary.

3. Take the equation of any curve in the xy-plane, e.g., the ellipse

$$2x^2 + y^2 = 1,$$

and plot all points (x, y, z) such that

$$2x^2 + y^2 = 1.$$

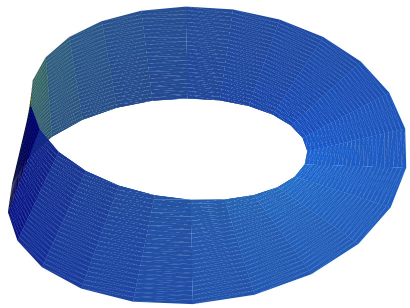
The result is a surface that is perpendicular to the xy-plane. This type of surface is called a (generalized) *cylinder*.

4. Surfaces of Revolution. Start with the graph of a function of one variable $z = f(y)$ in the yz-plane. Then we can revolve that curve around the y-axis in space. We get a surface of revolution.

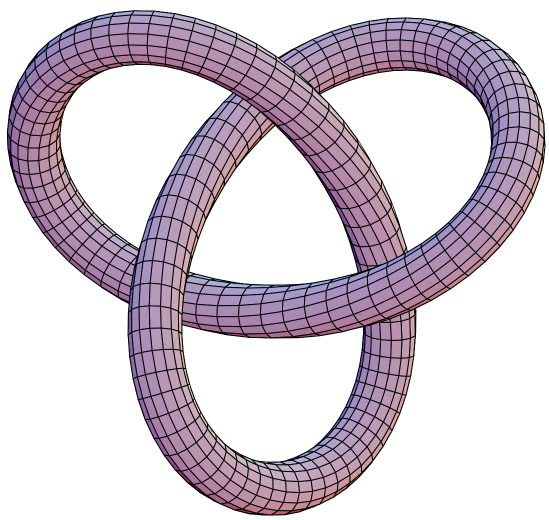
5. Graph of a function of two variables. Given a “nice” function $z = f(x, y)$, then the set of all points (x, y, z) such that $z = f(x, y)$ is a surface.

   Example. Consider the function $f(x, y) = x^2 + y^2$.

6. The Mobius Band. Take a long and relatively thin strip of paper and attach the two short ends by a half twist.



7. Surfaces from torus knots: This figure was produced using computer code posted on the web site of Professor Mark McClure at the University of North Carolina in Asheville.



8. Carin Siegerman surface: In class I will show a figure that can be interpreted as a surface. It was produced by Carin Siegerman, a former MA225 student of mine, as part of her work on her PhD dissertation in biomedical engineering.

9. Quadric Surfaces: surfaces of the form

$$Ax^2 + By^2 + Cz^2 = D$$

where A, B, and C are nonzero.