Quadric surfaces

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz + G = 0.$$

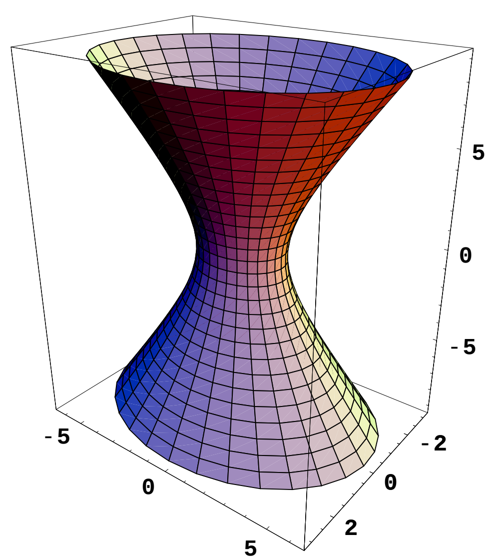
We start by studying the traces (cross sections) in various planes.

Example. Consider the surface

$$x^2 + \frac{y^2}{4} - \frac{z^2}{9} = 1.$$

Suppose we intersect the surface with the plane $z = k$.

(Additional white space for traces on the top of the next page.)


5
0
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2

Given any equation for a quadric surface, you need to be able to sketch it using the methods I described here. The slices are conic sections. You should know the names of the different types and what the differences are. There are six types.

1. Ellipsoid
2. Elliptic Paraboloid
3. Elliptic Cone
4. Hyperboloid of One Sheet
5. Hyperboloid of Two Sheets
6. Hyperbolic Paraboloid

See p. 682 of your textbook and the "Interactive Gallery of Quadric Surfaces" web site provided courtesy of Professor Jonathan Rogness of the University of Minnesota. (A link to that site is available in the classroom demos portion of our class web site.)

Parametric Surfaces

A parametrized surface in space is a set of points in space described by a (position) vector-valued function of the form

$$\mathbf{r}(u, v) = x(u, v)\,\mathbf{i} + y(u, v)\,\mathbf{j} + z(u, v)\,\mathbf{k},$$

where the functions $x(u, v)$, $y(u, v)$, and $z(u, v)$ are defined on some region in the uv-plane.

Example. The elliptic cylinder

$$x^2 + \frac{y^2}{4} = 1$$

is parametrized by the function $\mathbf{r}(\theta, z) = (\cos\theta)\,\mathbf{i} + (2\sin\theta)\,\mathbf{j} + z\,\mathbf{k}$.

One way to graph parametrized surfaces is to use the graphing applet provided by Professor Tom Leathrum of Jacksonville State University. A link to that applet is available in the classroom demos portion of our class web site.

Example. Parametrize the surface of revolution obtained by revolving the graph of

$$z = \frac{y^2}{10}(y^2 - 9) + 3$$

with $0 \le y \le 2.5$ around the y-axis.