IDE Worksheet #2

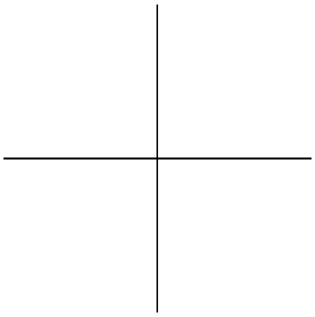
Name: ______________________________ Last five digits of ID Number: __________

Date: __________ Start Time: ________ End Time: ________ Monitor: ________

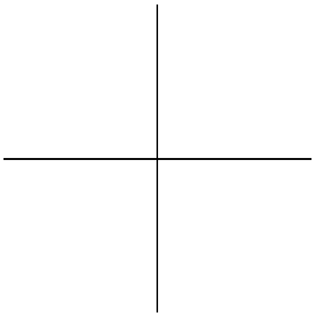
Goals: Use IDE to create a few phase portraits. Gain experience with Euler's method for systems. Study the relationship between solution curves in the phase plane and the corresponding $x(t)$- and $y(t)$-graphs.

1. The `Vector Fields` tool: This tool should remind you of the Slope Fields tool that you worked with on the last worksheet.

   (a) Consider the system $\dot{x} = 2y$, $\dot{y} = -x$. Plot several solutions to this differential equation and sketch your results below. Do enough solutions so that you get a general impression of the behavior of all solutions with initial conditions in the given window. (Make sure that you note the relationship between the solution curves and the vector field.)

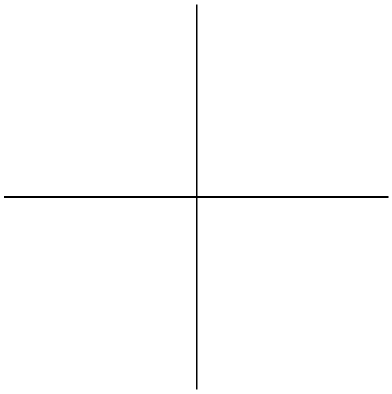


   (b) Consider the system $\dot{x} = y^2$, $\dot{y} = x$. Plot several solutions to this differential equation and sketch your results below. Do enough solutions so that you get a general impression of the behavior of all solutions with initial conditions in the given window. (Make sure that you note the relationship between the solution curves and the vector field.)



(c) Consider the system $\dot{x} = -y$ and $\dot{y} = x - y$. Use the tool with $\Delta t = 0.5$ to perform Euler's method graphically for the initial condition $(x_0, y_0) = (0, 2)$. Complete the following table and plot the points that you obtain. (Make sure that you label your axes and indicate the scales on both axes.)

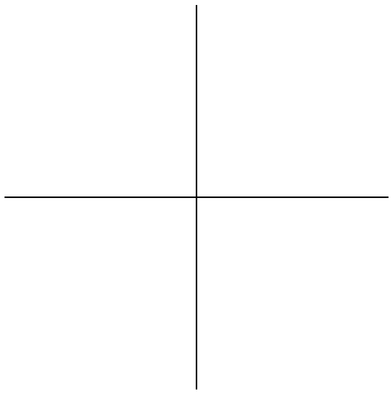
k	x_k	y_k
0	0	2.0
1		
2		
3		
4		
5		
6		



On the same axes above, plot the solution curve obtained by using the RK4 $\Delta t = 0.01$ numerical method.

Where is the vector field exactly horizontal with all of the vectors pointing to the right?

Plot several solutions to this differential equation and sketch your results below. Do enough solutions so that you get a general impression of the behavior of all solutions with initial conditions in the given window.



2. The `Phase Plane Drawing` Tool: In this tool, you draw in the phase plane by holding down the mouse, and the corresponding $x(t)$- and $y(t)$-graphs are (quickly!) generated by the tool.

   (a) Draw a circle centered at the origin in the phase plane. Then plot the corresponding $x(t)$- and $y(t)$-graphs that you obtain. Sketch the graph of $x(t)$ as a solid curve and the graph of $y(t)$ as a dashed curve.

   (b) Draw a square centered at the origin in the phase plane. Then plot the corresponding $x(t)$- and $y(t)$-graphs that you obtain. Sketch the graph of $x(t)$ as a solid curve and the graph of $y(t)$ as a dashed curve.

   (c) Draw a spiral tending toward the origin in the phase plane. Then plot the corresponding $x(t)$- and $y(t)$-graphs that you obtain. Sketch the graph of $x(t)$ as a solid curve and the graph of $y(t)$ as a dashed curve.

(d) Below are the $x(t)$- and $y(t)$-graphs for some "mystery" curve in the phase plane. Your mission ("if you choose to accept") is to sketch the corresponding curve in the phase plane. You'll probably need many attempts with the tool before you'll get anything close to the graphs given, but that's ok. (This is not so easy to do, so study the graphs closely before your initial attempt to sketch the curve.) Then sketch the curve below. Indicate the point at which you began and the direction you followed as you sketched out the solution curve. Make sure that you label the axes in both diagrams and label the $x(t)$- and the $y(t)$-graphs in the top figure.