IDE Worksheet #3

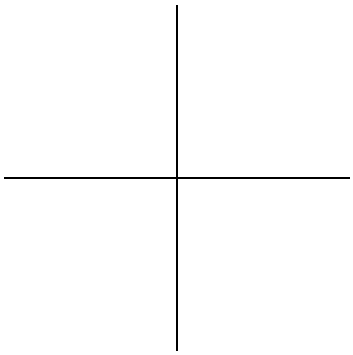
Name: _____ Last five digits of ID Number: _____

Date: _____ Start Time: _____ End Time: _____ Monitor: _____

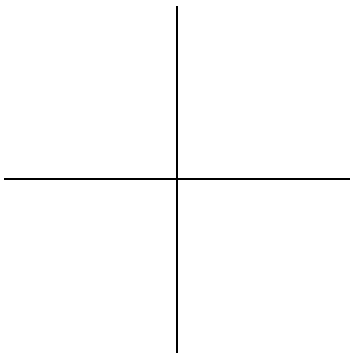
Goals: To understand the relationship between eigenvectors, eigenvalues, and solutions to linear systems. Examine the interpretation of these ideas for mass-spring systems. To understand the effects of periodic forcing in second-order, linear equations.

1. The `Eigen Engine` tool: We can find eigenvalues using the characteristic polynomial and eigenvectors by solving linear equations, but this tool illustrates the geometric significance of eigenvalues and eigenvectors without reference to the algebra.

   (a) Consider the matrix $\mathbf{A}$ that is associated to the linear system $\dot{x} = y$, $\dot{y} = -2x - 3y$. Using the tool, produce two linearly independent eigenvectors $\mathbf{V}_1$ and $\mathbf{V}_2$ and their associated eigenvalues. Sketch these vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ along with $\mathbf{AV}_1$ and $\mathbf{AV}_2$ on the axes given.

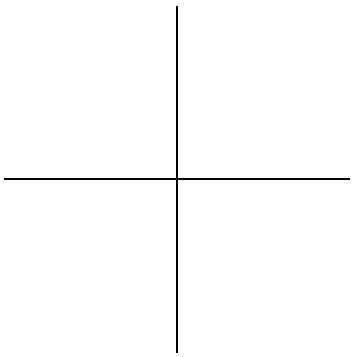


   (b) Consider the matrix $\mathbf{B}$ that is associated to the linear system $\dot{x} = y$, $\dot{y} = -x - 2y$. Using the tool, produce an eigenvector $\mathbf{V}$ and its associated eigenvalue. Can you find another eigenvector that is linearly independent from $\mathbf{V}$? Sketch $\mathbf{V}$ along with $\mathbf{BV}$ on the axes given.

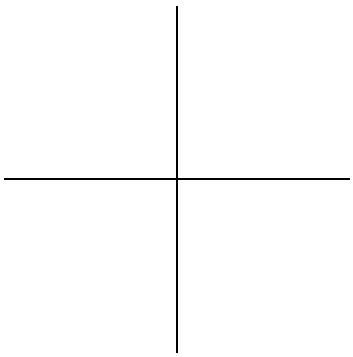


2. The `Matrix Element Input` tool: With this tool, you can enter a 2×2 matrix and sketch the corresponding phase portrait. Use this tool to sketch the phase portraits that correspond to the matrices **A** and **B** given earlier in the worksheet. Make sure that you illustrate the connection between eigenvalues and eigenvectors and the phase portrait. **You do not have to sketch the direction field**, but you should indicate the direction that the solution curves go in positive time.

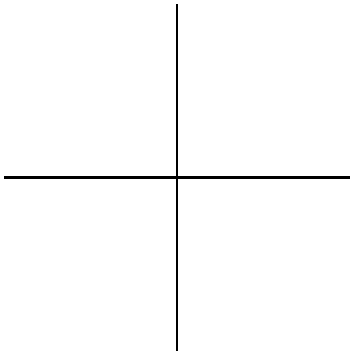
   (a) $\dfrac{d\mathbf{Y}}{dt} = \mathbf{AY}$



   (b) $\dfrac{d\mathbf{Y}}{dt} = \mathbf{BY}$

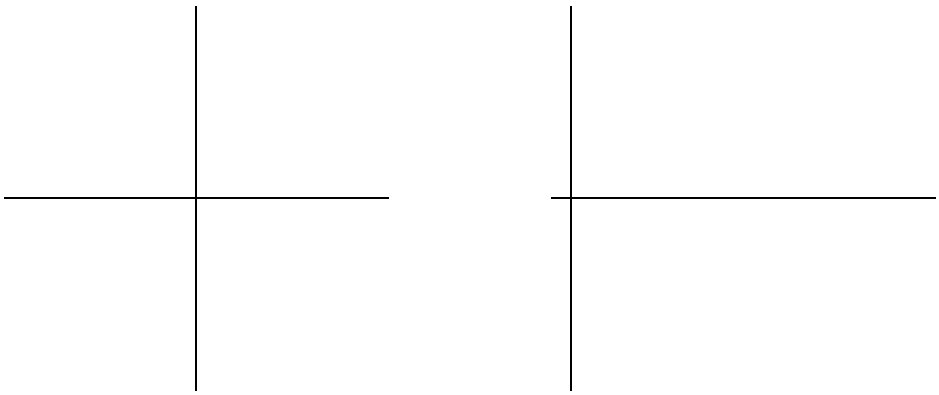


   (c) Now sketch the phase portrait for the system $\dot{x} = 1.9x - 2y$, $\dot{y} = 4x - 2.1y$.

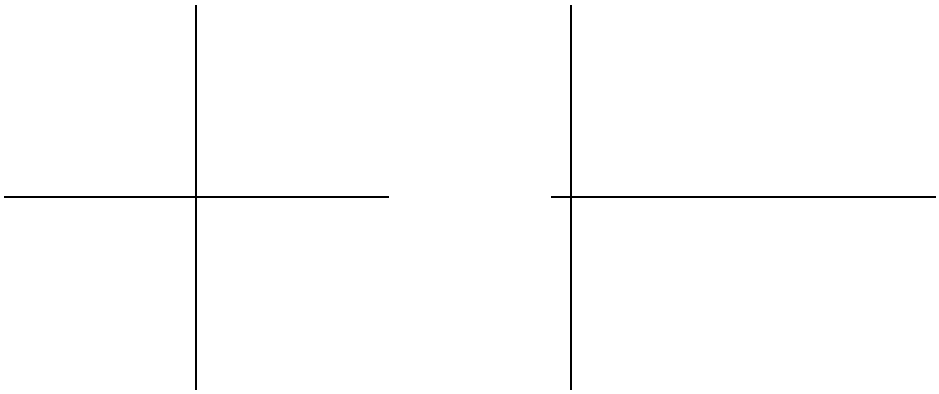


3. The `Mass and Spring` tool: This tool illustrates the motion of a mass-spring system given various values of the coefficients to the equation. For each choice of coefficients, provide a rough sketch of the direction field, the solution curve corresponding to the solution with initial condition $(\dot{x}_0, x_0) = (4, 0)$, the graph of the corresponding position function $x(t)$, and provide a brief description of the movement of the mass including its long-term behavior. Let the mass $m = 1$ throughout.

   (a) Spring constant $k = 1$ and damping constant $b = 0$:

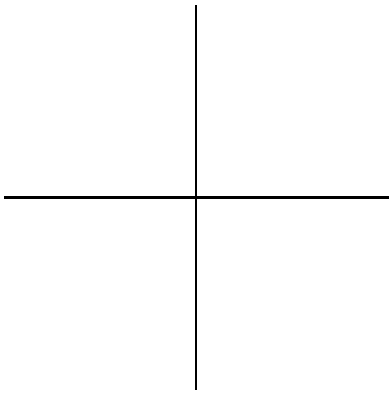


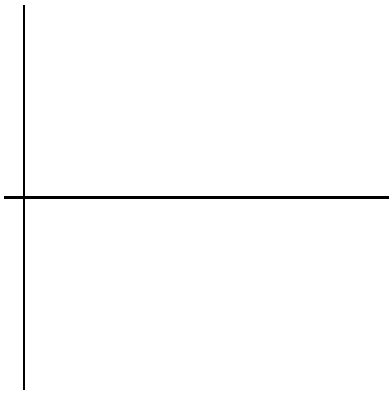
   (b) Spring constant $k = 2$ and damping constant $b = 0$:



   In what ways does this solution differ from the solution in part (a)?

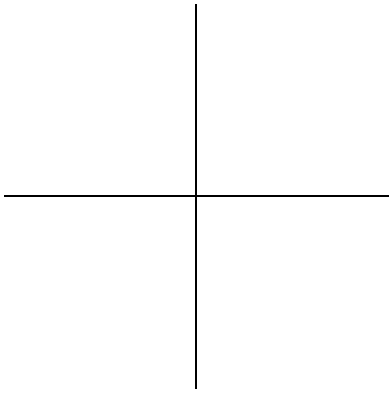
(c) Spring constant $k = 2$ and damping constant $b = 0.5$:

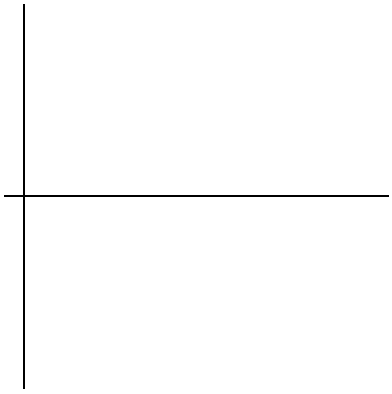




Does the mass ever come to a complete rest?

(d) Spring constant $k = 2$ and damping constant $b = 3$:

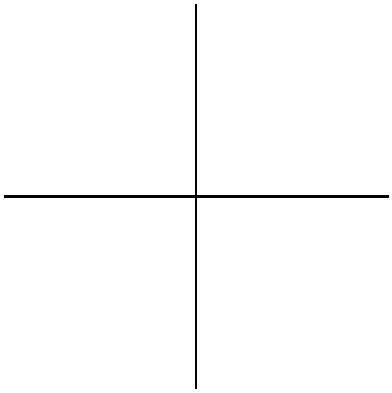


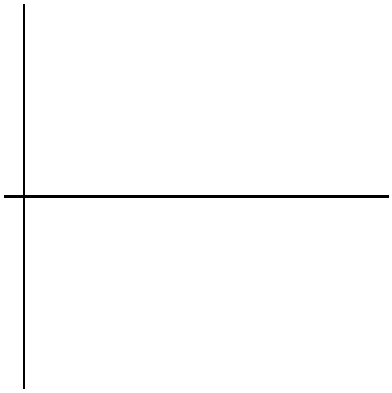


Which matrix, **A** or **B**, in Part 2 has the same portrait as this example?

4. The `Damped Forced Vibrations` tool: This tool illustrates the solutions of a forced, linear equation in terms of a mass-spring system with a dashpot.

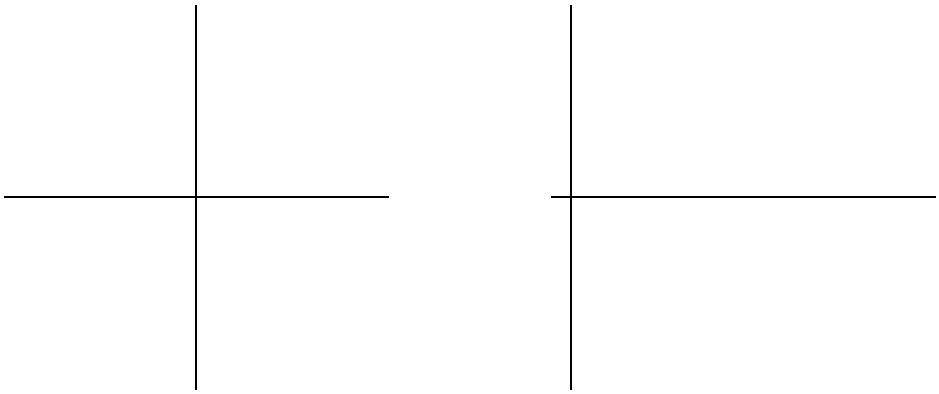
   (a) Start this worksheet with the default parameter values of $m = 1.5$, $b = 1$, $k = 1$, $F = 2$, and $\omega = 1$. Turn off the transient and steady state graphs, and plot the graph of the solution with initial condition $(x_0, \dot{x_0}) = (3, -3)$ (as close as you can get). You should also plot the corresponding solution curve in the phase plane.





   Give a brief description of the long-term behavior of this solution. How does this behavior manifest itself in both plots?

(b) Change the damping coefficient to 0.5 and turn on graphing of the transient. Plot the solution corresponding to the initial condition $(x_0, \dot{x_0}) = (3.0, -3.0)$ (as close as you can get).



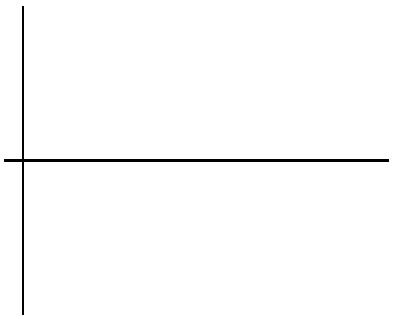
How do you know that this system is nonautonomous from these graphs alone?

(c) Turn off the forcing and compute the solution with the initial condition $(x_0, \dot{x_0}) = (2.8, 0.0)$. What happened to the graph of the transient? What calculation can you make that predicts the behavior of this solution?

(d) Turn off graphing the transient and return the value of the forcing to 2.0. Set the damping coefficient to 0, and plot the graph of the solution that corresponds to the initial condition $(x_0, \dot{x_0}) = (0, 0)$.

Note that this solution is commonly described as a "beating phenomenon."

(e) Turn on graphing the steady state and set the value of ω to 0.0. Plot the graph of the solution that corresponds to the initial condition $(x_0, \dot{x_0}) = (1, 0)$.



Give a brief explanation of the resulting graphs.

Those of you who are studying circuits may profit from a little experience with the `series circuits` tool. Note the similarity between the behaviors of the solutions in that tool and the behaviors that you observed on this worksheet.