IDE Worksheet #3

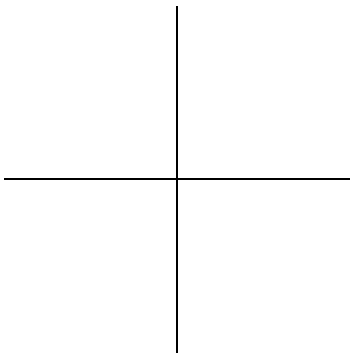
Name: _____ Last five digits of ID Number: _____

Date: _____ Start Time: _____ End Time: _____ Monitor: _____

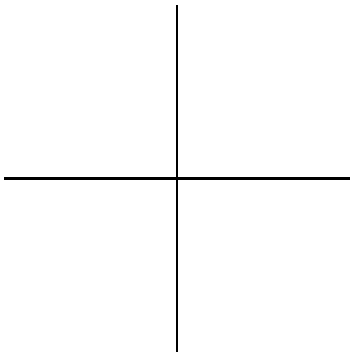
Goals: To understand the relationship between eigenvectors, eigenvalues, and solutions to linear systems. Examine the interpretation of these ideas for mass-spring systems.

1. The `Eigen Engine` tool: We can find eigenvalues using the characteristic polynomial and eigenvectors by solving linear equations, but this tool illustrates the geometric significance of eigenvalues and eigenvectors without reference to the algebra.

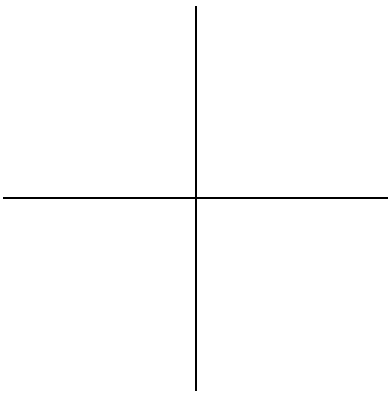
   (a) Consider the matrix $\mathbf{A}$ that is associated to the linear system $\dot{x} = y$, $\dot{y} = -2x - 3y$. Using the tool, produce two linearly independent eigenvectors $\mathbf{V}_1$ and $\mathbf{V}_2$ and their associated eigenvalues. Sketch these vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ along with $\mathbf{AV}_1$ and $\mathbf{AV}_2$ on the axes given.



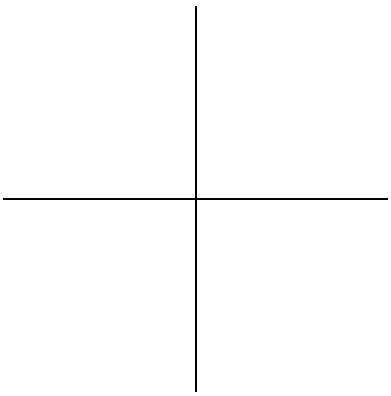
   (b) Consider the matrix $\mathbf{B}$ that is associated to the linear system $\dot{x} = y$, $\dot{y} = 2x$. Using the tool, produce two linearly independent eigenvectors $\mathbf{V}_1$ and $\mathbf{V}_2$ and their associated eigenvalues. Sketch these vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ along with $\mathbf{BV}_1$ and $\mathbf{BV}_2$ on the axes given. (Look carefully at the text on the screen and find a bug in this tool.)



(c) Consider the matrix $\mathbf{C}$ that is associated to the linear system $\dot{x} = y$, $\dot{y} = -x+3y$. Using the tool, produce two linearly independent eigenvectors $\mathbf{V}_1$ and $\mathbf{V}_2$ and their associated eigenvalues. Sketch these vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ along with $\mathbf{CV}_1$ and $\mathbf{CV}_2$ on the axes given.



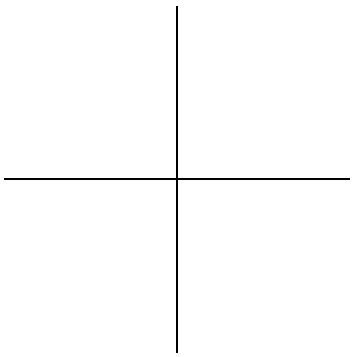
(d) Consider the matrix $\mathbf{D}$ that is associated to the linear system $\dot{x} = y$, $\dot{y} = -x-2y$. Using the tool, produce an eigenvector $\mathbf{V}$ and its associated eigenvalue. Can you find another eigenvector that is linearly independent from $\mathbf{V}$? Sketch $\mathbf{V}$ along with $\mathbf{DV}$ on the axes given.



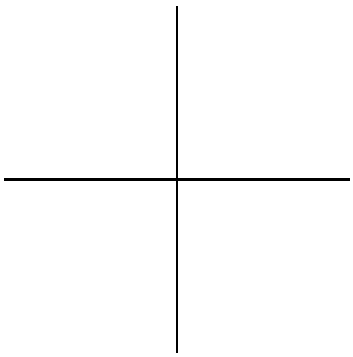
(e) Consider the matrix $\mathbf{E}$ that is associated to the linear system $\dot{x} = x - 2y$, $\dot{y} = x - y$. Using the tool, can you find eigenvectors and eigenvalues for $\mathbf{E}$? Why not?

2. The `Matrix Element Input` tool: With this tool, you can enter a 2×2 matrix and sketch the corresponding phase portrait. Use this tool to sketch the phase portraits that correspond to the matrices $\mathbf{A}$, $\mathbf{B}$, ..., $\mathbf{E}$ given earlier in the worksheet. Make sure that you illustrate the connection between eigenvalues and eigenvectors and the phase portrait. **You do not have to sketch the direction field**, but you should indicate the direction that the solution curves go in positive time.

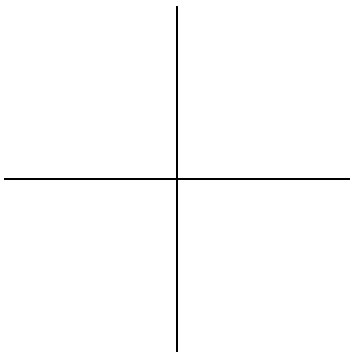
(a) $\dfrac{d\mathbf{Y}}{dt} = \mathbf{AY}$



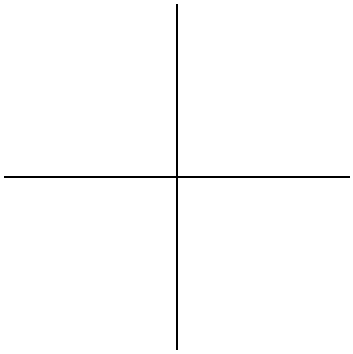
(b) $\dfrac{d\mathbf{Y}}{dt} = \mathbf{BY}$



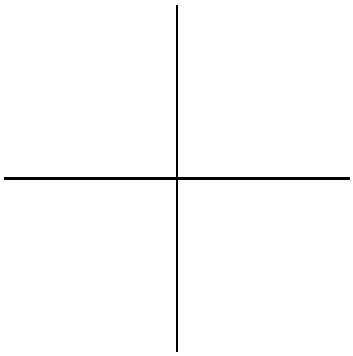
(c) $\dfrac{d\mathbf{Y}}{dt} = \mathbf{CY}$



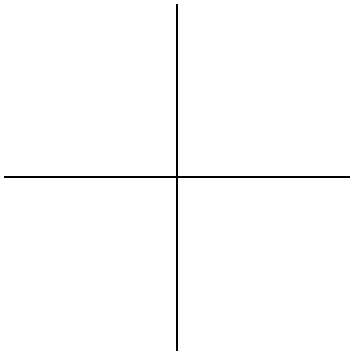
(d) $\dfrac{d\mathbf{Y}}{dt} = \mathbf{DY}$



(e) $\dfrac{d\mathbf{Y}}{dt} = \mathbf{EY}$

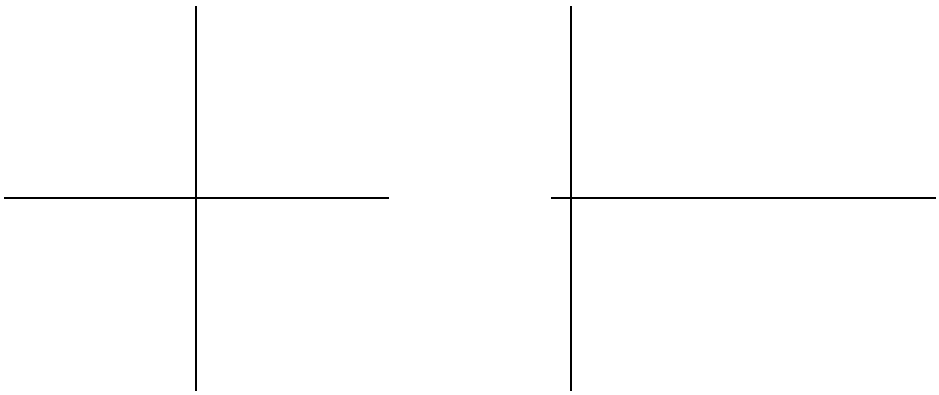


(f) Now sketch the phase portrait for the system $\dot{x} = 1.9x - 2y$, $\dot{y} = 4x - 2.1y$.

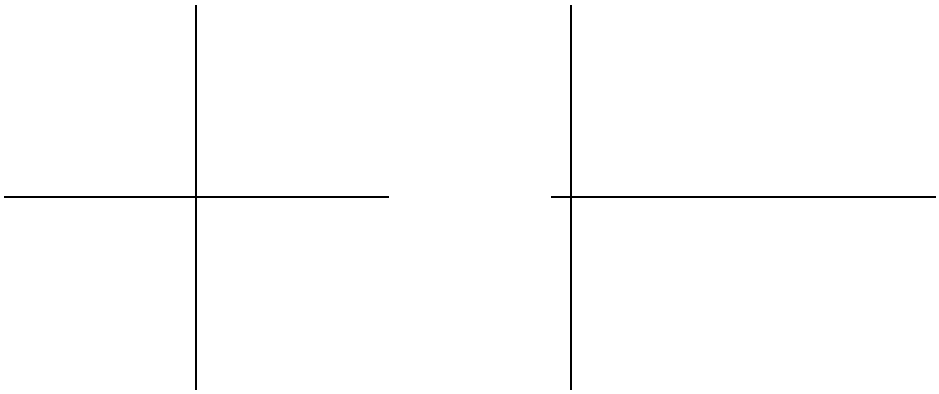


3. The `Mass and Spring` tool: This tool illustrates the motion of a mass-spring system given various values of the coefficients to the equation. For each choice of coefficients, provide a rough sketch of the direction field, the solution curve corresponding to the solution with initial condition $(\dot{x}_0, x_0) = (4, 0)$, the graph of the corresponding position function $x(t)$, and provide a brief description of the movement of the mass including its long-term behavior. Let the mass $m = 1$ throughout.

   (a) Spring constant $k = 1$ and damping constant $b = 0$:

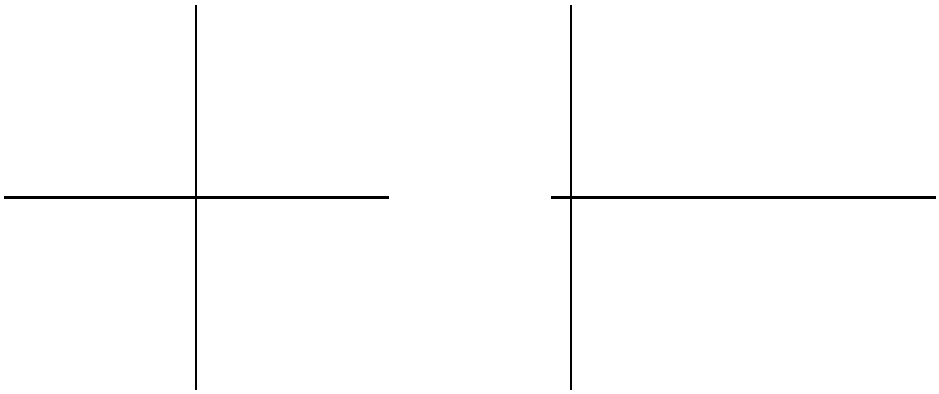


   (b) Spring constant $k = 2$ and damping constant $b = 0$:



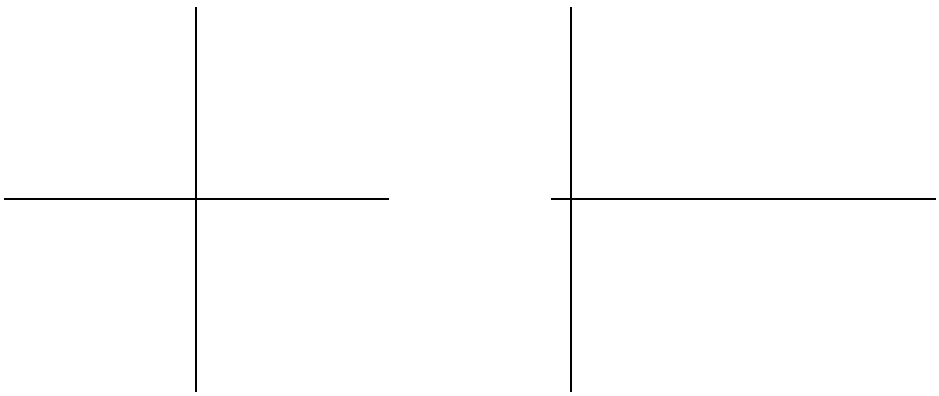
   In what ways does this solution differ from the solution in part (a)?

(c) Spring constant $k = 2$ and damping constant $b = 0.5$:



Does the mass ever come to a complete rest?

(d) Spring constant $k = 2$ and damping constant $b = 3$:



Which matrix, $\mathbf{A}$, $\mathbf{B}$, ..., $\mathbf{E}$, in Part 2 has the same portrait as this example?