MA 226 **November 4, 1999**

Glider Project

Due dates: There are two due dates associated with this project. The preliminary report is due at the start of class on November 23. The final report is due at the start of class on December 9.

Group project: Your group is limited to at most four members. Once a group begins work on the project, its membership cannot change. Consequently establishing your group must be your first step in this project. Each group will submit one report, and all members of the group will receive the same grade for this project.

Goals:

1. To apply what you have learned in this course to an example that is somewhat more complicated than the types of examples we discuss in class and study in the homework.
2. To get experience doing numerical work with differential equations.

The Glider: One of my favorite tools in IDE is the glider tool, which is based on the system of equations

$$\frac{dv}{dt} = -\sin\theta - Dv^2$$
$$\frac{d\theta}{dt} = \frac{v^2 - \cos\theta}{v}$$

where $v > 0$. The parameter D is a coefficient for the drag force on the glider. Also, note that the equations are periodic in θ. Actually these equations are not the true equations of motion but convenient "rescalings" of those equations.

Basic analysis: Start your analysis of this system by using the IDE glider tool. Identify the different types of motions of the glider both in the case of no drag ($D = 0$) and when drag is present ($D > 0$). What do the variables represent? Does the system have equilibrium points? What is the physical interpretation of the equilibria, both in the $D = 0$ and the $D > 0$ cases?

If $D = 0$, the tool also displays a value of a quantity that is denoted by E. In fact, $E = v^3 - 3v\cos\theta$. Using the Chain Rule, show that the value of E remains constant during any particular flight of the glider. (The function E is called a conserved quantity of the system.) What is the most negative value of E? What does the function E tell you about the solution curves of the system? Are the solutions periodic?

Now assume the presence of some drag ($0 < D < 4$) and use pplane to study the behavior of various solutions. What are the different possible behaviors of solutions? Are the solutions periodic? What eventually happens to the flight of the glider?

Linearization about the equilibria: Apply the theory of linearization to classify all equilibria for all values of D in the interval $0 \leq D \leq 4$, and determine the bifurcation values

of D. That is, determine the values of D where nearby D's lead to "different" behaviors in the phase portrait. For example, $D = 0$ is a bifurcation value because for $D = 0$ the long-term behavior of the solutions is dramatically different than the long-term behavior of the solutions if D is slightly positive. (*Hint:* If the type of an equilibrium point changes at a certain value of D, then that value of D may be a bifurcation value.)

Reconstructing the motion of the glider from the equations: Given a value of D and an initial condition (v_0, θ_0), then the the motion of the glider is determined from the equations. Show how one can start with values of D and (v_0, θ_0) and obtain the path of the glider. Be precise. (One good way to do this part of the project is to write the code that you would need to draw the path of the glider in some convenient programming language such as MATLAB.)

The phase cylinder: Why is it more natural to talk about the phase "cylinder" rather than the phase plane for this system? What changes if you analyze the system using a phase cylinder rather than a phase plane?

Your reports: Your preliminary report should be no longer than three typewritten pages. It should address the questions raised in the basic analysis section, and it should discuss how you are going to go about answering the "reconstruction" question. The preliminary report will be graded, and that grade will represent 20% of the final grade for the project.

Your final report should be no longer than six typewritten pages, and it should address all of the questions raised in the project. You may provide as many illustrations from the computer as you wish, but the relevance of each illustration to your report must be evident. Illustrations are not included in the six-page limit. (Please remember that, although one good illustration may be worth 1000 words, 1000 illustrations are worth nothing.)

Plagiarism: All work that you submit in this course must be your original work. In particular, the computations that you do for your report as well as the text of your report must be original to your group. All group members are responsible for all aspects of both reports.