MA 226 Sample Examinations

Attached are the four examinations that were given in my MA 226 class in the Spring of 2009. You should use them to get an idea of the format of a typical test and to see the types of questions I ask. *You should not assume that the test questions this semester will be on the same topics.* In fact, you are always responsible for *all* of the material that we cover in class as well as *all* of the designated material from your text, and the best way to study for my exams is to be sure that you are very comfortable with the homework assignments and the examples that I present in class. My tests often vary in difficulty (as you can see from the attached), and your grade for the examination will be determined by a curve that will be announced in class after the examination is graded.

MA 226	Exam 1A	February 11, 2009

Name: ______________________________ Last five digits of ID number: __________

Discussion Section (circle yours): M 12–1 M 2–3 M 3–4 T 10–11 T 11–12

Directions: Please do all of your work in this exam booklet and make sure that you cross out any work that we should ignore when we grade. **Books and extra papers are not permitted.** If you have a question about a problem, please ask. Remember: answers that are written logically and clearly will receive higher scores. There are 5 questions on 6 pages (not counting this cover page). Please make sure that you have all 6 pages of questions.

Do not write in the following box:

PROBLEM	POSSIBLE	SCORE
Name, etc.	3	
1	18	
2	20	
3	19	
4	20	
5	20	
TOTAL	100	

1. (18 points) Short answer questions: The answers to these questions need only consist of one or two sentences, but you do need to show enough work so that we can tell that you are not just guessing. Partial credit will be awarded only in exceptional situations.

   (a) Find all equilibrium solutions of the differential equation

   $$\frac{dy}{dt} = \frac{(t^2-4)(1+y)(y^2-2)e^y}{(t-1)(3-y)}.$$

   (b) Is $y(t) = t^2$ a solution to the differential equation $\dfrac{dy}{dt} = y^2 - t^2y + y - t^2 + 2t$? Why?

   (c) Give an example of a first-order, autonomous, linear, nonhomogeneous differential equation that has the equilibrium solution $y(t) = 3$ for all t.

2. (20 points) Consider the following 8 first-order equations:

1. $\dfrac{dy}{dt} = y^2 + y - 2$ 2. $\dfrac{dy}{dt} = y^2 - y - 2$ 3. $\dfrac{dy}{dt} = t^2 - 2$ 4. $\dfrac{dy}{dt} = t^2 - 2t$

5. $\dfrac{dy}{dt} = ty - t$ 6. $\dfrac{dy}{dt} = ty + t$ 7. $\dfrac{dy}{dt} = y + 2t$ 8. $\dfrac{dy}{dt} = y - 2t$

Four of the associated slope fields are shown on the next page. Pair the slope fields with their associated equations. Provide a brief justification for your choice. **You will not receive any credit unless you justify your selection.**

(a) The equation for slope field A is ____. My reason for choosing this answer is:

(b) The equation for slope field B is ____. My reason for choosing this answer is:

(c) The equation for slope field C is ____. My reason for choosing this answer is:

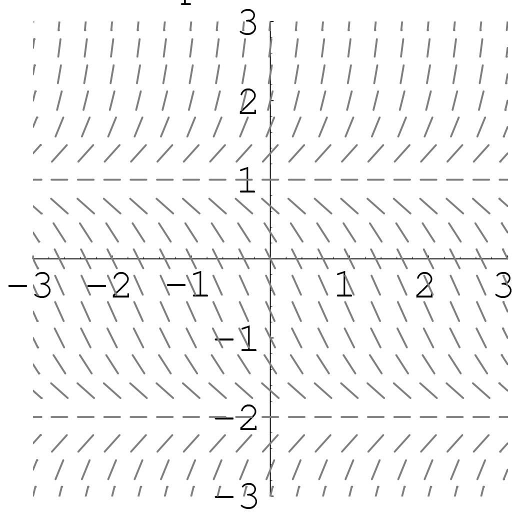
(d) The equation for slope field D is ____. My reason for choosing this answer is:

2. (continued) **Answer this question on the previous page.** The equations are provided here for your convenience:

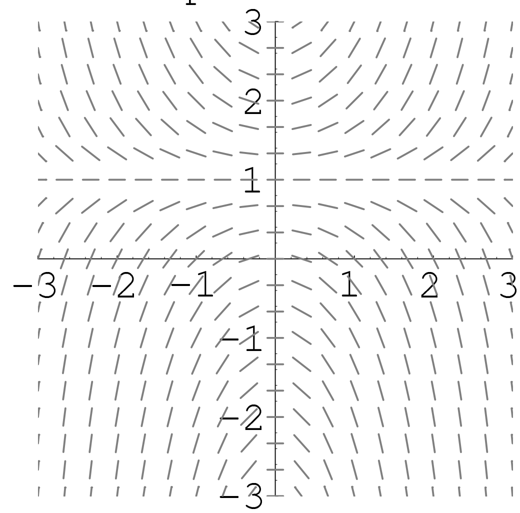
1. $\dfrac{dy}{dt} = y^2 + y - 2$ 2. $\dfrac{dy}{dt} = y^2 - y - 2$ 3. $\dfrac{dy}{dt} = t^2 - 2$ 4. $\dfrac{dy}{dt} = t^2 - 2t$

5. $\dfrac{dy}{dt} = ty - t$ 6. $\dfrac{dy}{dt} = ty + t$ 7. $\dfrac{dy}{dt} = y + 2t$ 8. $\dfrac{dy}{dt} = y - 2t$

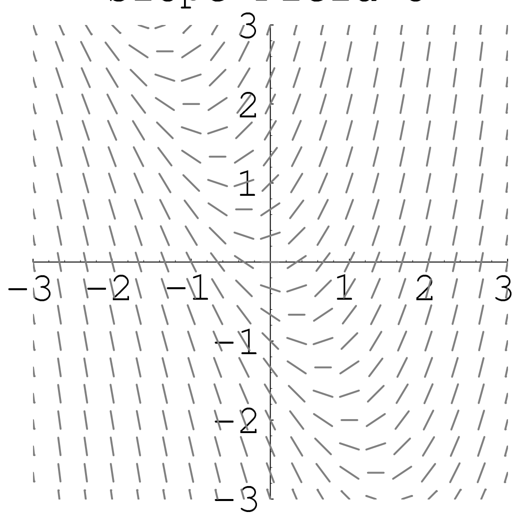
Slope Field A



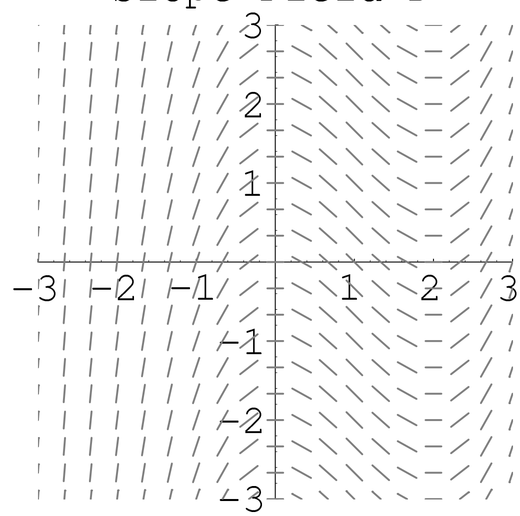
Slope Field B



Slope Field C



Slope Field D



3. (19 points) Calculate the general solution of the differential equation

$$\frac{dy}{dt} = 2y^2 - 4ty^2.$$

4. (20 points) Consider the initial-value problem

$$\frac{dy}{dt} = 3 - y^2, \quad y(0) = 0.$$

(a) Calculate the results of Euler's method applied to this initial-value problem on the interval $[0, 2]$ with 4 subdivisions. (Make sure that you show enough calculations so that we can see that you know the method.) Then graph your results. Make sure that you label the axes on your graph and clearly indicate the scale on each axis. You can use a calculator and do all calculations to 2 decimal places if you wish.

(b) Sketch the phase line for this differential equation. What does it tell you about your results in part (a)?

5. (20 points) The air in a small rectangular room 20 ft by 5 ft by 10 ft is 4% carbon monoxide. Starting at $t = 0$, air containing 1% carbon monoxide is blown into the room at the rate of 100 ft^3/min and well mixed air flows out through a vent at the same rate.

   (a) Write an initial-value problem that describes the amount of carbon monoxide in the room.

   (b) Sketch the phase line corresponding to the initial-value problem in part (a), and determine how much carbon monoxide will be in the room over the long term.

   (c) When will the air in the room be 2% carbon monoxide?

MA 226	Exam 2A	March 25, 2009

Name: ______________________ Last five digits of ID number: __________

Discussion Section (circle yours): M 12–1 M 2–3 M 3–4 T 10–11 T 11–12

Directions: Please do all of your work in this exam booklet and make sure that you cross out any work that we should ignore when we grade. **Books and extra papers are not permitted.** If you have a question about a problem, please ask. Remember: answers that are written logically and clearly will receive higher scores. There are 5 questions on 5 pages (not counting this cover page). Please make sure that you have all 5 pages of questions.

Do not write in the following box:

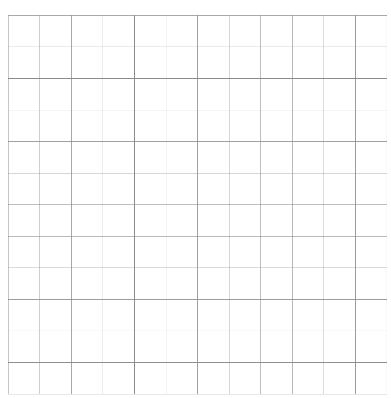
PROBLEM	POSSIBLE	SCORE
Name, etc.	3	
1	19	
2	20	
3	18	
4	20	
5	20	
TOTAL	100	

1. (19 points) Consider the linear system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} \mathbf{Y}.$$

(a) Determine the type (sink, saddle, source, ...) of the equilibrium point at the origin and find *all* straight-line solutions. Make sure that you show the computations that justify your answers.

(b) Sketch the phase portrait of this system over the square $-3 \leq x \leq 3$ and $-3 \leq y \leq 3$.



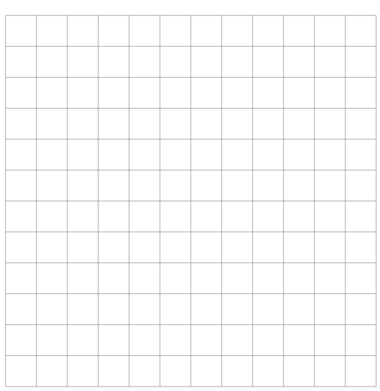
2. (20 points) Let $h(y) = y$ if $y > 0$ and $h(y) = 0$ if $y \leq 0$. Consider the initial-value problem

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y + 2h(y) = 1, \quad (y_0, v_0) = (-0.1, 0).$$

(a) Reduce this second-order equation to a first-order system and use Euler's method with 5 steps to calculate an approximate solution over the interval $0 \leq t \leq 1$. Enter the results in the table provided. Make sure that you show enough calculations so that the grader can understand how you obtained your answer. You may use a calculator and do all calculations to two decimal places if you wish.

k	y_k	v_k
0	-0.1	0
1		
2		
3		
4		
5		

(b) On the left-hand grid, sketch the approximate solution curve in the yv-phase plane, and on the right-hand grid, sketch the graphs of the approximations to the functions $y(t)$ and $v(t)$. Make sure that you put labels and scales on all axes.



solution curve in yv-plane

$y(t)$- and $v(t)$-graphs

3. (18 points) Short answer questions: The answers to these questions need only consist of one or two sentences, but you do need to show enough work so that we can tell that you are not just guessing. Partial credit will be awarded only in exceptional situations.

   (a) How many equilibrium points does the system $dx/dt = x(x - y)$ and $dy/dt = (x^2 - 4)(y^2 - 9)$ have? What are they?

   (b) Find the matrix $\mathbf{A}$ for which the function $\mathbf{Y}(t) = (3\cos 2t, \sin 2t)$ is a solution to $d\mathbf{Y}/dt = \mathbf{AY}$.

   (c) Is the function $(x(t), y(t)) = (e^{-6t}, 2e^{-3t})$ a solution to the system $dx/dt = 2x - 2y^2$ and $dy/dt = -3y$? Why?

4. (20 points) Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 5 & 3 \end{pmatrix}.$$

Its eigenvalues are $\lambda = 2 \pm 3i$. Compute the general solution to $d\mathbf{Y}/dt = \mathbf{A}\mathbf{Y}$.

5. (20 points) Solve the initial-value problem

$$\frac{dx}{dt} = x + 2y + 1 \qquad \qquad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}.$$
$$\frac{dy}{dt} = 3y$$

MA 226	Exam 3A	April 23, 2009

Name: ______________________________ Last five digits of ID number: ____________

Discussion Section (circle yours): M 12–1 M 2–3 M 3–4 T 10–11 T 11–12

Directions: Please do all of your work in this exam booklet and make sure that you cross out any work that we should ignore when we grade. **Books and extra papers are not permitted.** If you have a question about a problem, please ask. Remember: answers that are written logically and clearly will receive higher scores. There are 4 questions on 5 pages (not counting this cover page). Please make sure that you have all 5 pages of questions.

Do not write in the following box:

PROBLEM	POSSIBLE	SCORE
Name, etc.	3	
1	20	
2	24	
3	25	
4	28	
TOTAL	100	

1. (20 points) Solve the initial-value problem

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = 0, \quad y(0) = 4, \quad y'(0) = -11.$$

2. (24 points) Consider the system

$$\frac{dx}{dt} = 10 - x^2 - y^2$$
$$\frac{dy}{dt} = 3x - y.$$

(a) Determine all equilibrium points of this system.

(b) Identify the types of the equilibrium points that you found in Part a. In other words, determine if they are sinks, saddles, sources, ... Make sure that you indicate how you derived your answer.

3. (25 points)

(a) Calculate $\mathcal{L}^{-1}\left[\dfrac{1}{s^2-6s+8}\right]$.

(b) Compute $\mathcal{L}[u_4(t)\,t]$ **directly from the definition of the Laplace transform**.

4. (28 points) On the next page, there are eight second-order linear equations and four graphs of solutions. Match each graph with its corresponding equation. Provide a brief justification for your choice. **You will not receive any credit unless you provide a valid justification.**

   (a) The equation for graph A is ____. My reason for choosing this answer is:

   (b) The equation for graph B is ____. My reason for choosing this answer is:

   (c) The equation for graph C is ____. My reason for choosing this answer is:

   (d) The equation for graph D is ____. My reason for choosing this answer is:

4. (continued) **Answer this question on the previous page.**

The 8 second-order linear equations:

1. $\dfrac{d^2y}{dt^2} + 3y = 6$

2. $\dfrac{d^2y}{dt^2} + 5y = 10$

3. $\dfrac{d^2y}{dt^2} + 4y = -8$

4. $\dfrac{d^2y}{dt^2} + 3y = \cos 9t$

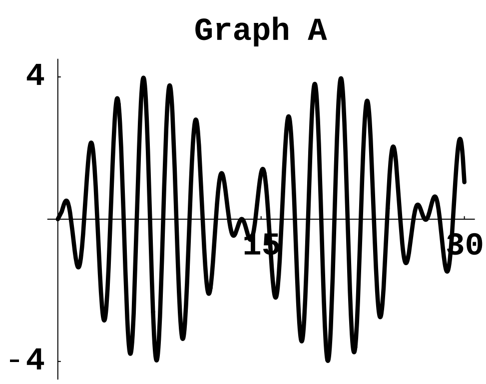
5. $\dfrac{d^2y}{dt^2} + 9y = \cos 3t$

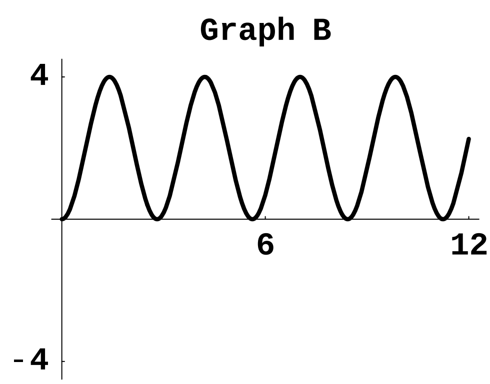
6. $\dfrac{d^2y}{dt^2} + 12y = 6\cos 3t$

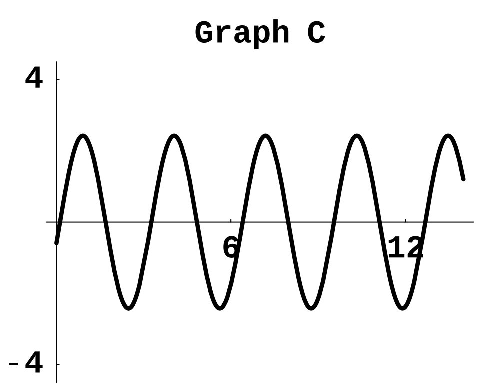
7. $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + 3y = 10\cos 2t$

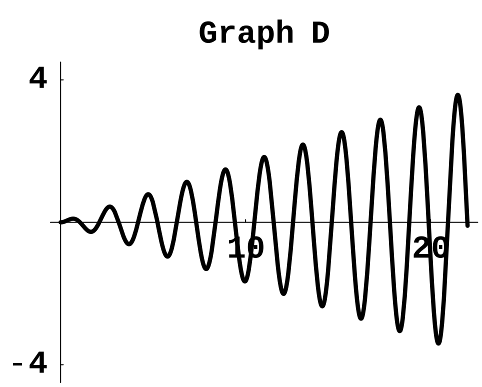
8. $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + 3y = \cos 2t$

The four graphs of solutions:









MA 226 | Final A | May 7, 2009

Name: ______________________ Last five digits of ID number: __________

Directions: Please do all of your work in this exam booklet and make sure that you cross out any work that we should ignore when we grade. **Books and extra papers are not permitted.** If you have a question about a problem, please ask. Remember: answers that are written logically and clearly will receive higher scores. There are 7 questions on 9 pages (not counting this cover page). Please make sure that you have all 9 pages of questions.

Do not write in the following box:

PROBLEM	POSSIBLE	SCORE
1	16	
2	15	
3	15	
4	12	
5	12	
6	15	
7	15	
TOTAL	100	

1. (16 points) **Note that parts c and d of this problem are on the next page.** Consider the second-order equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 20y = -5\cos 5t.$$

   (a) What can you say about the long-term behavior of solutions without solving for the general solution? Be as specific as possible.

   (b) Determine a particular solution to this differential equation.

1. (continued) Here is the differential equation from the previous page:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 20y = -5\cos 5t.$$

(c) Find the general solution to this differential equation.

(d) What can you say about the long-term behavior of the solutions given your results from parts b and c? Be as specific as possible.

2. (15 points) Solve the initial-value problem

$$\frac{dx}{dt} = 4x - 2y$$

$$\frac{dy}{dt} = x + y$$

with $(x(0), y(0)) = (-1, -2)$.

3. (15 points) **Note that part c of this problem is on the next page.**

(a) Calculate $\mathcal{L}^{-1}\left[\dfrac{2s+5}{s^2+2s+3}\right]$.

(b) Calculate the Laplace transform $\mathcal{L}[y]$ for the solution $y(t)$ to the initial-value problem

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = \delta_4(t), \quad y(0) = 2, \quad y'(0) = 1.$$

DO NOT CALCULATE A FORMULA FOR $\mathbf{y(t)}$ HERE.

3. (continued)

(c) Calculate the solution $y(t)$ to the initial-value problem in part b.

4. (12 points) Short answer questions: The answers to these questions need only consist of one or two sentences. Partial credit will be awarded only in exceptional situations.

   (a) Find one solution of the differential equation $dy/dt = ty + 3y - 2t - 6$.

   (b) Sketch the solution curve for the initial-value problem $dx/dt = -2x$, $dy/dt = -2y$, and $(x_0, y_0) = (2, 1)$.

   (c) Find all equilibrium solutions of the equation $d^2y/dt^2 + 4y = \sin 2t$.

5. (12 points) Are the following statements true or false? **You must justify your answers to receive any credit.**

   (a) Every solution of $dy/dt = y + e^{-t}$ tends either to $+\infty$ or to $-\infty$ as $t \to \infty$.

   (b) The function $\mathbf{Y}(t) = (\cos 2t, \sin t)$ is not a solution to any linear system.

   (c) If the function $(x_1(t), y_1(t)) = (\cos t, \sin t)$ is a solution of a first-order autonomous system, then the function $(x_2(t), y_2(t)) = (-\sin t, \cos t)$ is also a solution of the same system.

6. (15 points) Consider the one-parameter family of linear systems

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} a & 1 \\ a & a \end{pmatrix} \mathbf{Y}.$$

(a) Sketch the curve in the trace-determinant plane that is obtained by varying the parameter a.

(b) Determine all bifurcation values of a and briefly discuss the different types of phase portraits that are exhibited in this one-parameter family.

7. (15 points) A can of soda just removed from the refrigerator is 50°. After 10 minutes of sitting in a 70° room, its temperature is 54°.

   (a) Assume that Newton's Law of "Cooling" applies: The rate of change of temperature is proportional to the difference between the current temperature and the ambient temperature. Write an initial-value problem that models the temperature of the can of soda.

   (b) Sketch the phase line corresponding to the initial-value problem in part (a), and determine the temperature of the can of soda over the long term.

   (c) How long does it take for the can of soda to reach 60°?