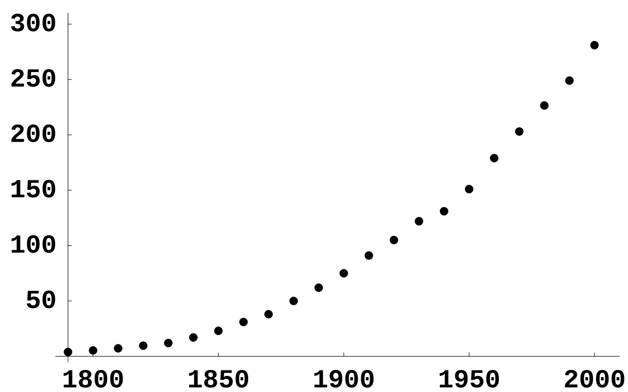
Modeling the US Population:
The data graphed as a function of time



Steps in Model Building

1. State underlying assumptions.
2. Identify the relevant variables and parameters.
3. Use the assumptions in Step #1 to formulate equations relating the variables in Step #2.

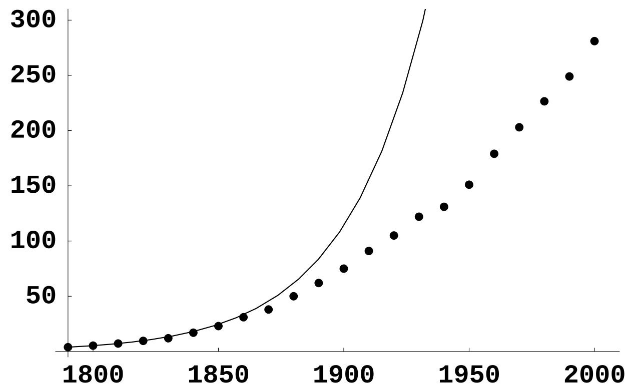
First Model: Malthusian Model
Assumption: Growth rate of the population is proportional to the population.
Variables: independent variable t for time (in years since 1790) and dependent variable p for population (in millions)
Malthusian model is

$$\frac{dp}{dt} = kp,$$

where k is a proportionality constant (a parameter).
The function

$$p(t) = p_0 e^{kt}$$

is a solution to this differential equation. Note that $p(0) = p_0$. The constant p_0 is called the initial condition of $p(t)$.
Using the fact that $p(0) = 3.9$ and $p(10) = 5.3$, we have $p_0 = 3.9$, and we can calculate $k = 0.03067$.
Here's the graph of $p(t)$ superimposed on the data:



Second Model: Logistic Model
Assumptions:

1. If the population is small, its growth rate is proportional to the size of the population.
2. As the population increases, its **relative growth rate** decreases.

What is a relative growth rate?

A Qualitative Analysis of the Logistic Model
We now have

$$\frac{dp}{dt} =$$

Can we determine the long-term behavior of solutions without computing the solutions first?

A Numerical Simulation of the Logistic for the US Population

If we want to study this model numerically, we need estimates for k and N. How do we approximate the relative growth rates from the data?