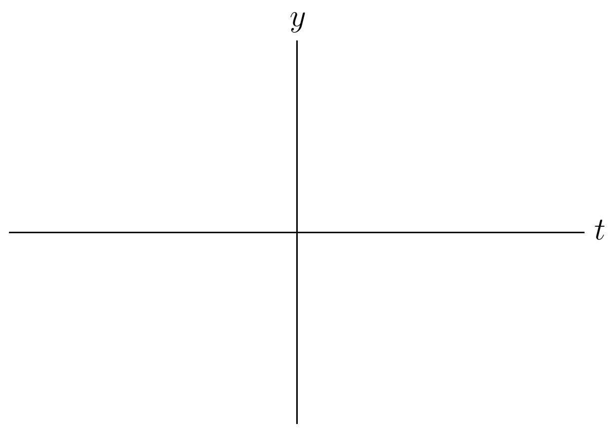
Euler's method:
Euler's method is the most basic of all of the numerical algorithms that are used to approximate solutions to differential equations. Let's start with an example to get an idea of how the method works.

Example. Consider the initial-value problem

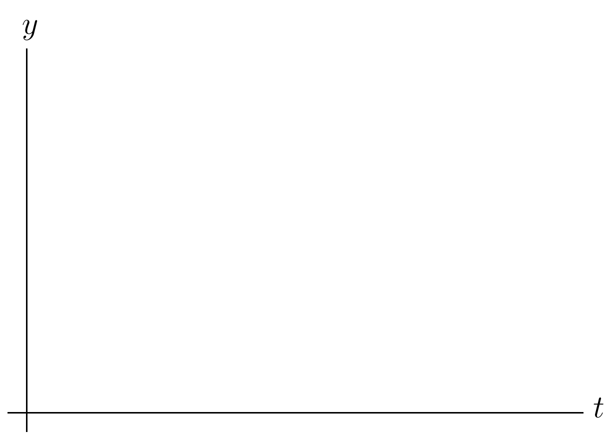
$$\frac{dy}{dt} = y^2 - t, \quad y(-1) = -1.$$

Let's see what happens when we use Euler's method to approximate the solution with a step size of $\Delta t = 0.5$. We'll use the `EulersMethod` tool from `DETools`.

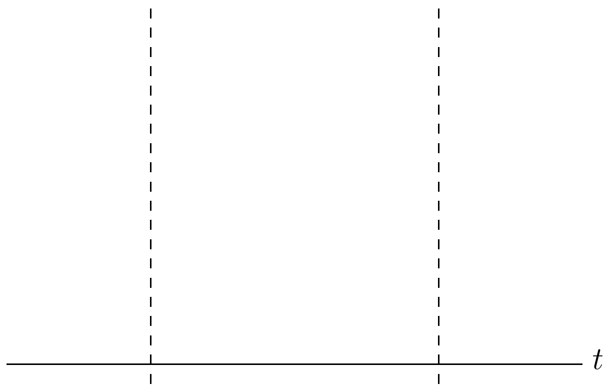
k	t_k	y_k
0	-1	-1
1		
2		
3		
4		
5		
6		

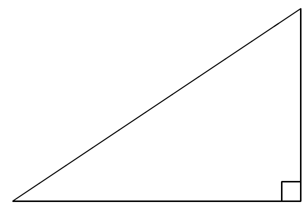


Now here is a general picture of the algorithm and the associated notation:



Let's look more closely at the kth step and the key triangle:





These observations yield Euler's method:

Euler's method is easy to program—even with just a spreadsheet.

	A	B	C	D	E	F	G
0	-1	-1		0.5			
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							
17							
18							

There are two spreadsheets posted on the course web site—one that uses a lot of the defaults in Excel and one that has been customized for the particular example we are discussing. The second spreadsheet has a slider for Δt that illustrates what happens when a different values of Δt are used.

Euler's method is the most basic "fixed step size" algorithm for numerically approximating solutions. `HPGSolver` also uses a fixed step size algorithm called the Runge-Kutta method. The Runge-Kutta method is usually more efficient and more accurate than Euler's method (see Chapter 7 of our text). Unfortunately, there are differential equations that are not amenable to fixed step size algorithms.

Example. Consider the initial-value problem

$$\frac{dy}{dt} = e^t \sin y, \quad y(0) = 5.$$

We'll see what happens when we use Euler's method to approximate the solution with various step sizes $0.01 \leq \Delta t \leq 0.1$. The spreadsheet for this example is also posted on the course web site.