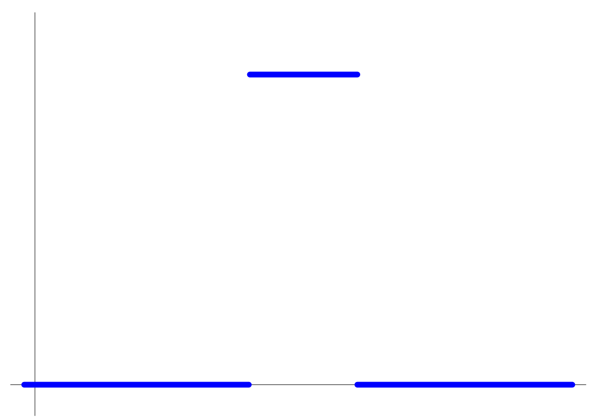
Impulse forcing

It is convenient to use Laplace transforms to solve linear differential equations with discontinuous forcing, but there are other, less convenient, ways to solve those equations. Today we consider equations that cannot be solved without use of transforms (as far as I know).

Dirac Delta Function. The Dirac delta "function" $\delta_a(t)$ is used to model impulse forcing. Suppose we want to model a unit force that is applied instantaneously at time $t = a$. We begin with the function

$$g_{\Delta t}(t) = \begin{cases} \dfrac{1}{\Delta t}, & \text{if } a - \dfrac{\Delta t}{2} \le t < a + \dfrac{\Delta t}{2}; \\ 0, & \text{otherwise.} \end{cases}$$



Then

$$g_{\Delta t} =$$

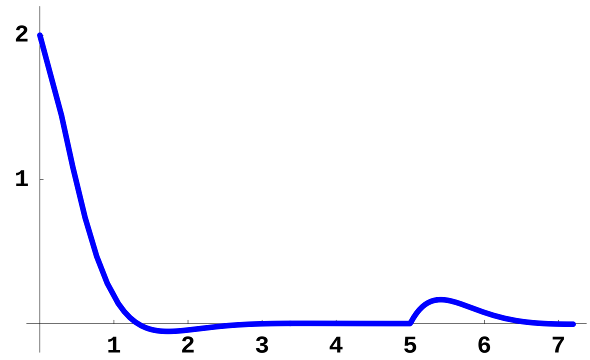
Dirac Delta Function. The Dirac delta function $\delta_a(t)$ is the "function" such that

$$\mathcal{L}[\delta_a] = e^{-as}.$$

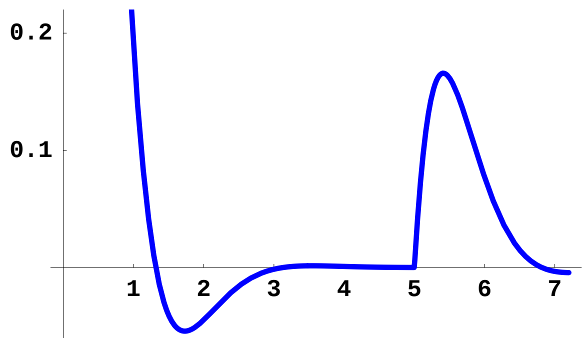
Example. Consider the initial-value problem

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 7y = \delta_5(t), \quad y(0) = 2,\ y'(0) = -1.$$

Here is the graph of the solution:



We enlarge the scale on the vertical axis:



Here is the graph of its derivative: