Simple mass-spring system:

Last class we derived the equation

$$m\frac{d^2y}{dt^2} + ky = 0$$

using Hooke's law. Let's consider the special case where $k = m$. We get

$$\frac{d^2y}{dt^2} = -y,$$

and we can guess some solutions to this equation:

In what ways is the mass-spring system similar to the predator-prey system?

An initial condition for the predator-prey system is a pair (R_0, F_0) of population values.

An initial condition for the mass-spring system is also a pair (y_0, v_0). The first number indicates the initial displacement and the second number indicates the initial velocity.

We can perform a mathematical reduction to the second-order equation for the mass-spring system so that it resembles the predator-prey system:

Some terminology:

- initial condition:

- graphs of solutions (sometimes called component graphs):

- solution curve in the phase plane:

- phase portrait:

General 2D autonomous systems

In general, a 2D first-order autonomous system of ordinary differential equations has

- one independent variable and
- two dependent variables.
- The independent variable does not appear on the right-hand sides of the differential equations.

Example. The glider

$$\frac{dy}{dt} = \frac{-\cos y + v^2}{v}$$
$$\frac{dv}{dt} = -\sin y - Dv^2$$

where D is a drag parameter.

First let's see if this system has any equilibrium solutions:

Terminology:

- initial condition:

- solution to an initial-value problem:

- component graphs:

- phase plane:

- solution curve in the phase plane:

- phase portrait:

One skill that you will learn is how to make a rough sketch of the component graphs from the solution curve. There is a tool on your CD called `DESketchPad` which will help you practice.