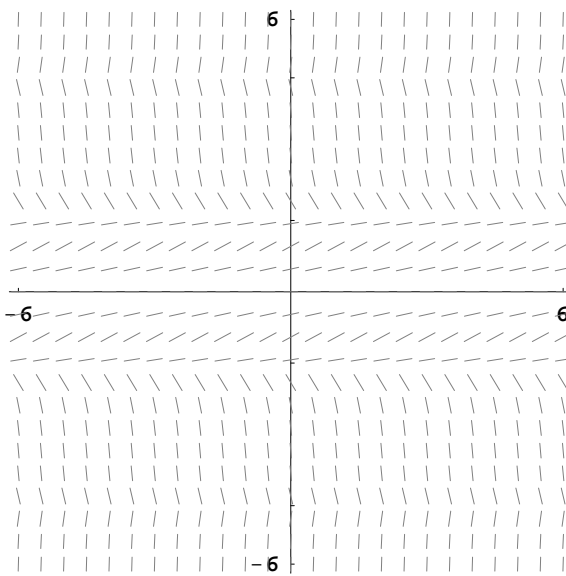


Building phase lines:

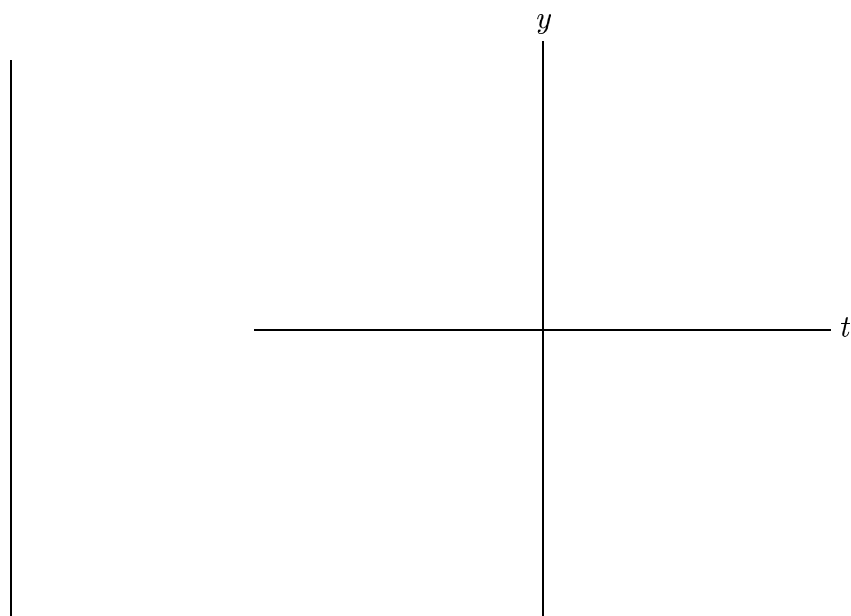
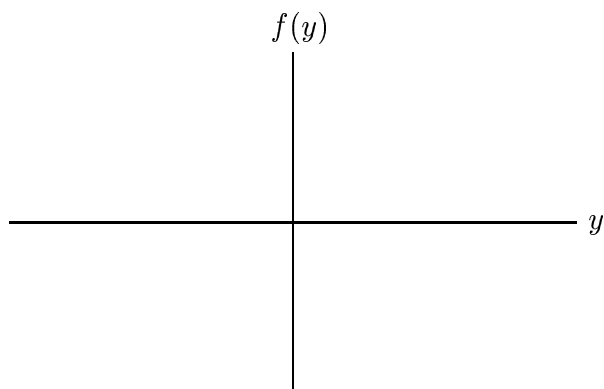
Given an autonomous equation $dy/dt = f(y)$, you need to know how to build the phase line from the formula for $f(y)$ and from the graph of $f(y)$.

Example 1. $\frac{dy}{dt} = y^2 \cos y$

First we build the phase line:



Example 2. $\frac{dy}{dt} = f(y)$ where $f(y)$ is given by the graph



Parameters, Qualitative Equivalence, and Bifurcations

Let's return to the logistic model of population growth

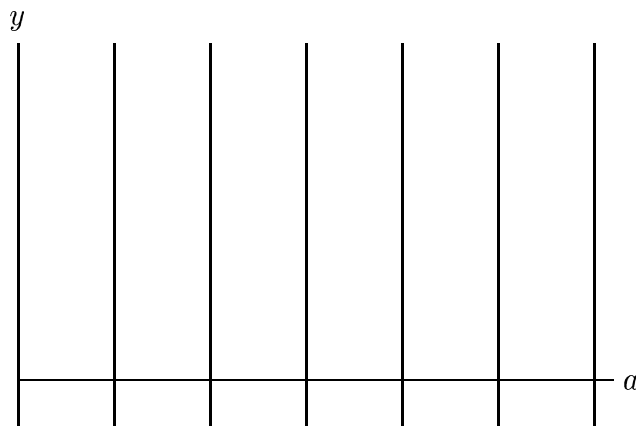
$$\frac{dP}{dt} = kP \left(1 - \frac{P}{N} \right)$$

and modify this model to account for constant harvesting:

Example. $\frac{dy}{dt} = y(1 - y) - a$

There is a tool in `DETools` called `PhaseLines`, and it helps us analyze phase lines and various graphs as we vary certain parameters (the parameter a in this case).

We can summarize the behavior of this one-parameter family of differential equations using the bifurcation diagram.



Now let's sketch and interpret the bifurcation diagram for the logistic population model with constant harvesting

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{N}\right) - C.$$

