Facts about eigenvalues and eigenvectors: Given a 2×2 matrix $\mathbf{A}$,

1. The characteristic equation can have two real roots, one real root of multiplicity two, or two complex conjugate roots.
2. Given an eigenvector $\mathbf{Y}_0$ associated to an eigenvalue λ, then any nonzero scalar multiple $\mathbf{Y}_0$ is also an eigenvector associated to λ.
3. Eigenvectors associated to distinct eigenvalues are linearly independent.

Summary of Case of Two Distinct Real Eigenvalues

Suppose $\mathbf{A}$ is a matrix with two eigenvalues λ_1 and λ_2. To be consistent, we will assume that $\lambda_1 < \lambda_2$, that $\mathbf{V}_1$ is an eigenvector associated to λ_1, and that $\mathbf{V}_2$ is an eigenvector associated to λ_2. The general solution of

$$\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$$

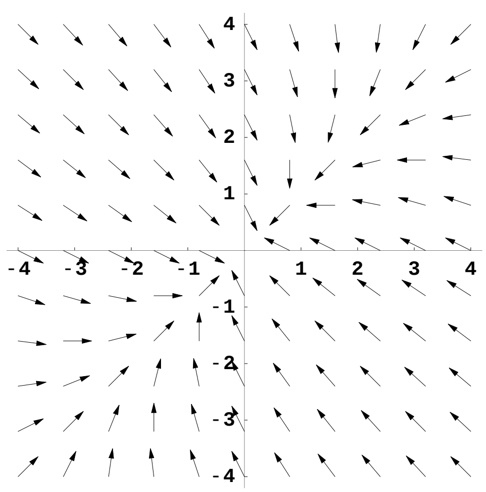
is

$$\mathbf{Y}(t) = k_1 e^{\lambda_1 t}\mathbf{V}_1 + k_2 e^{\lambda_2 t}\mathbf{V}_2.$$

Case 1: $\lambda_1 < \lambda_2 < 0$.

Example. Consider

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \mathbf{Y}.$$


-4
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x
t

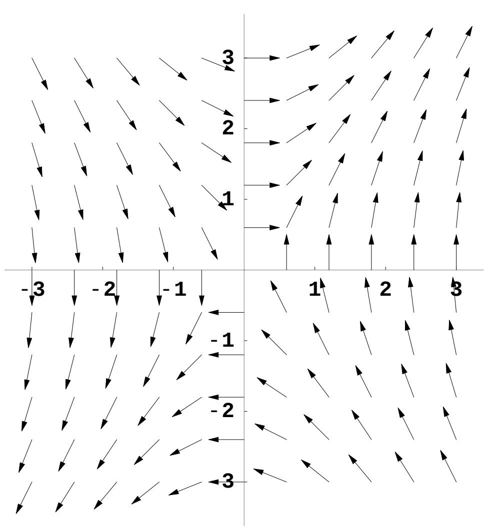

y
t

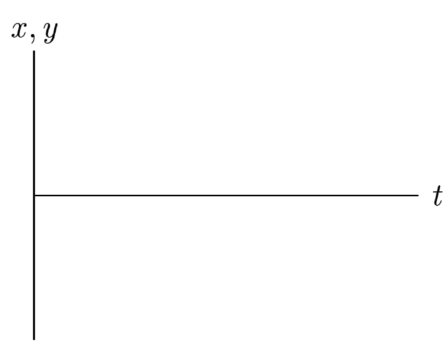
Case 2: $\lambda_1 < 0 < \lambda_2$.

Example. Consider

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix} \mathbf{Y}.$$

There are two tools on the CD that you can use to help understand this example—`HPGSystemSolver` and `LinearPhasePortraits`.





Case 3: $0 < \lambda_1 < \lambda_2$.

Example. Consider

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{Y}.$$