

At the end of last class, I made two general comments about first-order differential equations

$$\frac{dy}{dt} = f(t, y)$$

and their solutions.

1. I gave a rough description of what a differential equation is and what it means to solve an **initial-value** problem.
2. I mentioned two examples to emphasize the fact that the distinction between the independent and the dependent variables is important.

Example 1. $\frac{dy}{dt} = kt$

The solutions to this equation are

$$y(t) = k\frac{t^2}{2} + c,$$

where c is an arbitrary constant.

Example 2. $\frac{dy}{dt} = ky$

The solutions to this equation are

$$y(t) = y_0 e^{kt},$$

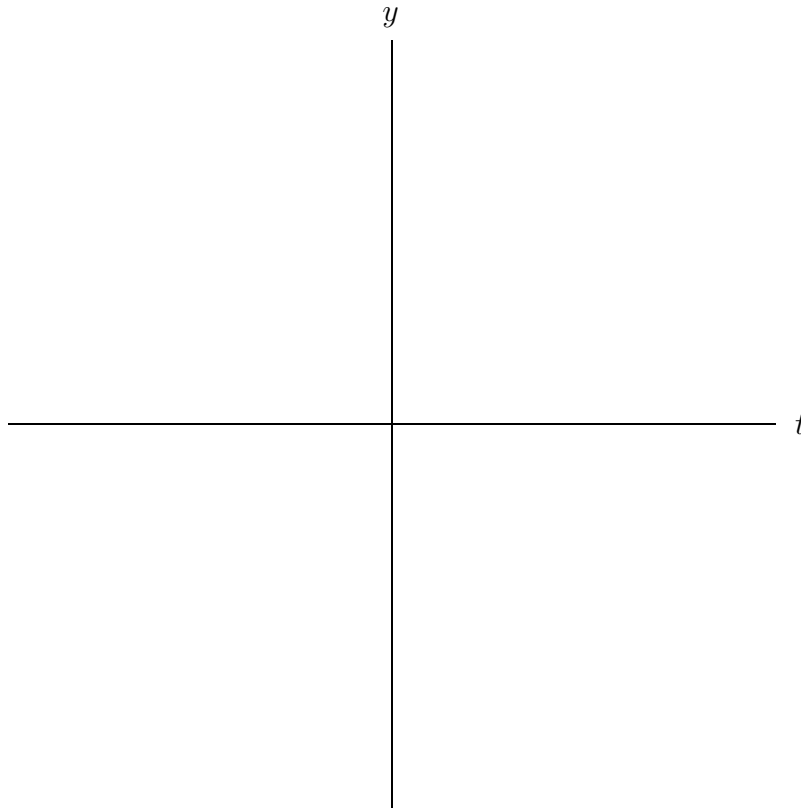
where y_0 is an arbitrary constant.

I have three additional comments:

3. What does the term **general solution** mean?

4. Why you should never get a wrong answer in this course.

5. Even relatively simple looking differential equations can have solutions that cannot be expressed in terms of functions that we already know and love.



Our general approach in this course:

We will study differential equations

1. using the theory and
2. various techniques:
 - (a) analytic techniques
 - (b) geometric/qualitative techniques, and
 - (c) numerical techniques.

Separable Differential Equations (an analytic technique)

First let's recall the method of substitution for calculating integrals (really antiderivatives):

A differential equation

$$\frac{dy}{dt} = f(t, y)$$

is **separable** if it can be written in the form

$$\frac{dy}{dt} =$$

Two Examples:

1. $\frac{dy}{dt} = -2ty^2$

2. $\frac{dy}{dt} = y^3 + t^2$

Let's go back to the first example

Example. $\frac{dy}{dt} = -2ty^2$

We turn to `HPGSolver` to get a sense of the graphs of these solutions:

