Method of the lucky guess

Last class we discussed how various solutions of a given linear equation are related. For homogeneous linear differential equations, the Linearity Principle says that solutions are multiples of one another. For nonhomogeneous linear equations, solutions satisfy the Extended Linearity Principle. We can paraphrase the Extended Linearity Principle by saying that:

> The general solution of a nonhomogeneous linear equation consists of the sum of *any* particular solution of the nonhomogeneous equation and the general solution of the associated homogeneous equation.

Consider the two examples we computed last class:

Example. $\dfrac{dy}{dt} = y - t.$

We calculated the general solution $y(t) = 1 + t + ce^t$.

Example. $\dfrac{dy}{dt} = -2ty + 4e^{t^2}.$

We calculated the general solution $y(t) = 4te^{-t^2} + ce^{-t^2}$.

The Extended Linearity Principle provides an additional way to solve nonhomogeneous equations:

Method of the Lucky Guess:

1. Solve the associated homogeneous equation.
2. Guess one solution to the nonhomogeneous equation.

How do we go about guessing a solution? This question is best answered by doing a number of typical examples.

Example 1. $\dfrac{dy}{dt} = y + 3e^{-t}$

1. General solution of the associated homogeneous equation:

2. Particular solution of the nonhomogeneous equation:

Example 2. $\dfrac{dy}{dt} = -2y + \cos 3t$

1. General solution of the associated homogeneous equation:

2. Particular solution of the nonhomogeneous equation:

Example 3. $\dfrac{dy}{dt} = -2y + 3e^{-2t}$

1. General solution of the associated homogeneous equation:

2. Particular solution of the nonhomogeneous equation (trick question):

Now we have two methods for solving nonhomogeneous linear equations. How do we decide which method to use?

Method of integrating factors:

1. In theory, it always works.
2. But you must calculate two integrals.

Method of the lucky guess:

1. Tends to be faster than the method of integrating factors—mostly algebra.
2. Note that the examples that we considered were all of the form

$$\frac{dy}{dt} = g(t)y + r(t)$$

   where $g(t)$ was a constant function ("constant-coefficient case"). What types of functions $r(t)$ did we consider?

3. But you need to be lucky.

Good problem to think about: How can you solve our old favorite

$$\frac{dy}{dt} = y - t$$

using a guessing technique?

MA 226 Exam Logistics

1. Bring pen/pencil and id. You may use your calculator if you wish. If you have a calculator that does symbolic derivatives/integrals, you should make sure that your answer shows that you could do the problem without your calculator.
2. Closed book exam. No extra papers.
3. Exam will start promptly at 10:00 and end at 10:50.
4. We will collect exams by moving up the aisles. You must pass in your exam when we arrive at your aisle. Please remain seated and quiet until we collect the exams from your aisle.
5. Five minute rule will be in effect: No one will be allowed to leave the exam between 10:45 and 10:50. Use those 5 minutes to check your work. (You should never get a wrong answer in this course.)
6. Seating will be assigned before the exam starts.
7. If you have a question, raise your hand. Stay seated.
8. Go to the bathroom before the exam starts.