Last class we discussed two examples—the predator-prey system and the mass-spring system.

Predator-prey:

$$\frac{dR}{dt} = aR - bRF$$
$$\frac{dF}{dt} = -cF + dRF$$

where the parameters a, b, c, and d are positive.

Mass-spring:

$$m\frac{d^2y}{dt^2} + ky = 0$$

where m is the mass and k is the spring constant.

Now I would like to introduce some terminology related to these two examples.

General 2D autonomous systems

In general, a 2D first-order autonomous system of ordinary differential equations has

- one independent variable and
- two dependent variables.
- The independent variable does not appear on the right-hand sides of the differential equations.

Keep the predator-prey example in mind as we go through some terminology:

- initial condition:
- solution to an initial-value problem:
- equilibrium solutions:

- component graphs:

- phase plane:

- solution curve in the phase plane:

- phase portrait:

One skill that you will learn is how to make a rough sketch of the component graphs from the solution curve. There is a tool on your CD called `DESketchPad` which will help you practice.

Mass-spring system: The simple mass-spring system is the second-order autonomous differential equation

$$m\frac{d^2y}{dt^2} + ky = 0.$$

In what ways is the mass-spring system similar to the predator-prey system?

An initial condition for the predator-prey system is a pair (R_0, F_0) of population values.

An initial condition for the mass-spring system is also a pair (y_0, v_0). The first number indicates the initial displacement and the second number indicates the initial velocity.

We can perform a mathematical reduction to the second-order equation for the mass-spring system so that it resembles the predator-prey system:

Last class we were able to guess two solutions to the mass-spring system in the case where $k = m$ because the equation can be rewritten as

$$\frac{d^2y}{dt^2} = -y$$

if $k = m$. We guessed

$$y_1(t) = \sin t \qquad \text{and} \qquad y_2(t) = \cos t.$$

The equivalent system

$$\begin{aligned}\frac{dy}{dt} &= v\\ \frac{dv}{dt} &= -y\end{aligned}$$

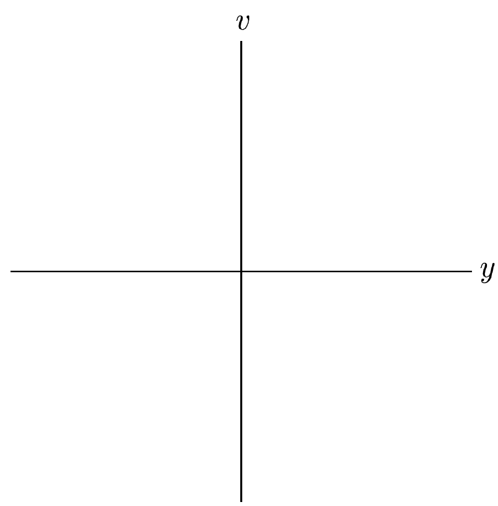
has one $(y(t), v(t))$ pair of solutions for each solution to the second-order equation:

What are the initial conditions for these solutions?

What are the component graphs?

What are the solution curves?

What is the phase portrait for the mass-spring system if $k = m$?



Where do we go from here?

1. Vector fields (similar to slope fields)
2. Two analytic techniques (more guessing)
3. Euler's method again
4. Some theory (Existence and Uniqueness)
5. Linear systems and equations (Chapter 3)

The vector field of an autonomous system

We get a better geometric understanding of the solutions of a first-order system if we convert the system into a vector equation.

Example 1. Once again we consider a simple mass-spring system

$$\frac{dy}{dt} = v$$
$$\frac{dv}{dt} = -y.$$

By some intelligent guessing we know a few solutions. One is

$$(y(t), v(t)) = (\cos t, -\sin t).$$

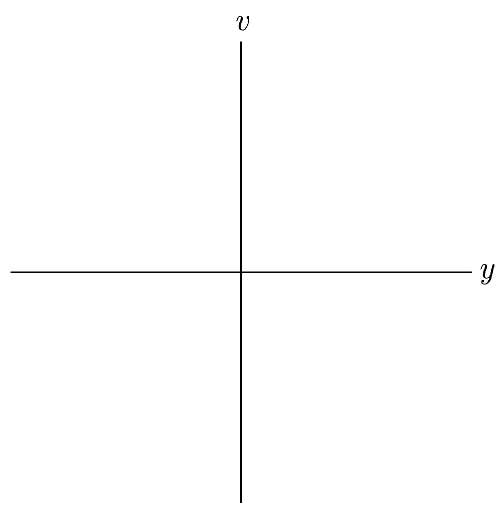
We rewrite this system and the solution in terms of vectors:

Now for the geometric interpretation of

$$\frac{d\mathbf{Y}}{dt} = \mathbf{F}(\mathbf{Y}),$$

where

$$\mathbf{F}(\mathbf{Y}) = \mathbf{F}\begin{pmatrix} y \\ v \end{pmatrix} = \begin{pmatrix} v \\ -y \end{pmatrix}.$$



The direction field associated with this system is